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The relationship between a declining labor income share and a falling relative price of capital requires capital and labor to be gross substitutes at the aggregate level (i.e., ???? > 1). I argue that this restriction can be relaxed if we distinguish labor by skills and identify differential capital-labor substitutability across skill groups. Using the Morishima elasticity of substitution in a three-factor nested-CES production function, I analytically estimate the elasticity of substitution parameters between capital and skilled labor ( ? ) and between capital and unskilled labor (?). I then derive the necessary conditions for a decline in the labor income share based on ? and ?, which does not require ???? to be greater than unity.

**Keywords**

substitution elasticity, labor income share, production function parameters

**Comments**

**Suggested Citation**

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# CAPITAL SKILL SUBSTITUTABILITY AND THE LABOR INCOME SHARE: IDENTIFICATION USING THE MORISHIMA ELASTICITY OF SUBSTITUTION

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## Abstract

The relationship between a declining labor income share and a falling relative price of capital requires capital and labor to be gross substitutes at the aggregate level (i.e., $\sigma_{Agg} > 1$). I argue that this restriction can be relaxed if we distinguish labor by skills and identify differential capital-labor substitutability across skill groups. Using the Morishima elasticity of substitution in a three-factor nested-CES production function, I analytically estimate the elasticity of substitution parameters between capital and skilled labor ($\rho$) and between capital and unskilled labor ($\sigma$). I then derive the necessary conditions for a decline in the labor income share based on $\rho$ and $\sigma$, which does not require $\sigma_{Agg}$ to be greater than unity.



**Keywords:** substitution elasticity, labor income share, production function parameters

**JEL Classification:** E21, E22, E25

# Contents

# 1. INTRODUCTION

Research in recent years (Elsby, Hobijn, and Sahin, 2013; Karabarbounis and Neiman, 2014; Piketty, 2014; Piketty and Zucman, 2014) has documented a global decline in the labor income share (LIS, here on)[1]. According to the "accumulation view" (Rognile, 2015), a decline in the LIS is driven by a fall in the relative price of capital goods (Karabarbounis and Neiman, 2014) or a rise in the stock of capital relative to income through a growth in aggregate savings (Piketty, 2014). In the presence of heterogeneous labor with capital-skill complementarity (Krusell et al., 2000), the conditions required for the "accumulation view" can be written as $\sigma > \rho > 1$, where $\sigma$ and $\rho$ stand for the elasticity of substitution between capital and unskilled labor and capital and skilled labor, respectively (Arpaia, Perez, and Pichelmann, 2009; Karabarbounis and Neiman, 2014). The condition $\sigma > \rho > 1$ implies that the aggregate substitution elasticity between capital and labor ($\sigma_{Agg}$) is greater than unity[2]. Both Piketty (2014) and Karabarbounis and Neiman (2014) estimate $\sigma_{Agg}$ to be greater than unity, which is at odds with extant literature predominantly estimating $\sigma_{Agg}$ to be less than one (Leon-Ledesma, McAdam, and Willman, 2015; Oberfield and Raval, 2014; Chirinko and Mallick, 2017)[3]. To this extent, with $\sigma_{Agg}$ < 1, the "accumulation view" mechanisms predict a rise in the LIS[4].

Two recent papers attempt to decipher this mystery. Rognile (2015) supports the "scarcity view", which assumes an increase in capital share due to the relative scarcity of some forms of capital, as opposed to the "accumulation view". When considering a multisector model with different types of capital, he distinguishes between the ${\sigma_{Agg}}^{NET}$ (the relationship between net capital-output ratio and net rental rate of capital) and ${\sigma_{Agg}}^{GROSS}$ (the standard definition) by highlighting the role of depreciation. He demonstrated that ${\sigma_{Agg}}^{NET} < {\sigma_{Agg}}^{GROSS}$ , ${\sigma_{Agg}}^{GROSS} > 1$ does not necessarily imply ${\sigma_{Agg}}^{NET} > 1$; as a result, following the "scarcity view," a decline in the LIS with a higher capital-output ratio can be attained with ${\sigma_{Agg}}^{NET} < 1$. In another paper, Grossman et al. (2017) used human capital accumulation in a standard neoclassical growth framework, and defined the three elasticity parameters, $\sigma_{HC,PC}$ (between human capital and physical capital) as $\sigma_{HC,RL}$ (between human capital and raw labor) and $\sigma_{L,PC}$ (between total labor and physical capital). This shows that if $\sigma_{HC,PC} < \sigma_{HC,RL}$, and $\sigma_{L,PC} < 1$, then a constant level of schooling would mean that movement in the share of labor as a proportion of national income and the rate of labor productivity growth would be positively correlated across steady states. This way, a decrease in labor productivity

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1 Other notable contributions to this topic, among others, are Bentolila and Saint-Paul (2003), Blanchard and Giavazzi (2003), Gollin (2002).

2 A non-unitary elasticity of substitution (σ) between capital and labor plays a crucial role in explaining movements in the labor income share. The role of σ in analyzing the factor income shares has been noted since the seminal work of Hicks (1932) and Robinson (1933). Following the Hicksian partial elasticity of substitution, Elsby, Hobijn, and Sahin (2013) demonstrate the relationship between labor income share ($L_S$) and σ as $dlnL_S = -(1 - L_S)\frac{\sigma-1}{\sigma}dln(\frac{K}{L})$, suggesting a drop in $L_S$ when σ > 1. With σ = 1, factor income shares remain constant.

3 Using sectoral level data, Herrendorf, Herrington, and Valentinyi (2015) show elasticity of substitution to be 1.58 in agriculture, .80 in manufacturing and .75 in services.

4 If capital and labor are gross complements, then a decline in capital per unit of effective labor can lead to an excessive increase in the rental rate compared to wages. This mechanism can lead to a lower income share of labor (Alvarez-Cuadrado, Long, and Poschke, 2015).

growth would correspond to a drop in the LIS. Grossman et al shows that a decline in the LIS is feasible with $\sigma_{Agg}$ < 1 if labor productivity growth slows down[5].

In this paper, I prove analytically that a decline in the LIS would be associated with a fall in the relative price of capital if $\sigma_{Agg} < 1$ (i.e., capital and labor as gross complements) in a production structure with the labor market being segmented by skill level. I draw insights from the micro-level estimation of the $\sigma_{Agg}$ (Oberfield and Raval, 2014) and the literature on differential capital-skill substitutability (Krusell, Ohanian, Rios-Lull, and Violante, 2000; Karabarbounis and Neiman, 2014). I then construct a theoretical framework that identifies the elasticities of substitution parameters between capital and labor across different skill-groups (in a nested-CES production structure) with the help of the Morishima elasticity of substitution (MES, here on). I then derive conditions under which a decline in the LIS would be associated with a fall in the relative price of capital with $\sigma_{Agg} < 1$.

To build a solid foundation, let us consider a segmented labor market and differential capital-labor substitutability across different skill levels (Griliches, 1969; Krusell, Ohanian, Rios-Lull, and Violante, 2000 [KORV, here on]). If capital is more substitutable with unskilled labor than skilled labor (Griliches, 1969; Berman, Bound, and Grilliches, 1994), then a drop in the share of income is likely to be larger for the unskilled labor than for the skilled labor, thus resulting in a drop in the relative price of capital[6]. A direct estimate of the $\sigma_{Agg}$ using the aggregate labor provides a weighted average of the elasticities of substitution between capital and labor across different skill groups; for this reason, it masks the role of differential capital-skill substitutability. Karabarbounis and Neiman (2014) addressed this concern by considering a modified version of the multi-input nested-CES production function discussed in KORV. With the three inputs of capital (K), skilled labor (S), and unskilled labor (U), the CES production function can be nested in three ways: $Y = f[(K,S)U]$, $Y = f[(K,U)S]$, and $Y = f[(S,U)K]$ (nested-inputs are within the first bracket). Since $Y = [(S,U)K]$ boils down to a standard 2-factor CES production, Karabarbounis and Neiman (2014) considered the other two functions in order to examine the link between capital-skill complementarity and the labor share of income. I write $Y = f[(K,S)U]$ as

$$Y = f[(K,S)U] = \left[\theta\left[\emptyset K^{\frac{\rho-1}{\rho}} + (1-\emptyset)S^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\frac{\sigma-1}{\sigma}} + \ (1-\theta)U^{\frac{\sigma-1}{\sigma}}\ \right]^{\frac{\sigma}{\sigma-1}} {}^{7} \tag{1}$$

From this production function, Karabarbounis and Neiman (2014) derived the following equation to estimate σ:

$$\frac{L_S}{1-L_S}\widehat{L_S} = \alpha + (\sigma-1)\hat{\delta} + \beta\widehat{\left(\frac{S}{K}\right)} + \varepsilon^{8}. \tag{2}$$

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5 To demonstrate the mechanism, Grossman et al. (2017) considered a drop in the interest rate relative to the growth rate of wages, which prompts individuals to achieve a higher level of human capital for any steady-state level of technology and the size of capital stock. Since, $\sigma_{HC,PC} < \sigma_{HC,RL}$ (i.e., human capital is more complementary to physical capital than raw labor) this generates a shift in the relative factor demand in favor of a rise in the capital income share.

6 This is also related to the large literature on skill-biased technical change (SBTC). See Grilliches (1969), Acemoglu (2002), Autor, Levy, and Murnane (2003), Caselli (1999), among others.

7 $\theta$ and $\emptyset$ denote distribution parameters; $\sigma$ denotes the elasticity of substitution between K and U (similarly, between U and S); $\rho$ denotes the elasticity of substitution between K and S.

Equation (2) suggests that a positive relationship between trends in the LIS and changes in the relative price of investment goods (compared to skilled labor) is only possible when σ > 1.

However, Equation (2) cannot identify the value of $\rho$ (the elasticity of substitution between nested inputs) or distinguish it from σ. Since the estimate of $\beta$ is a function of both $\sigma$ and $\rho$, equation 2 alone cannot identify the value of both elasticities of substitution[9]. Karabarbounis and Neiman (2014) estimated σ using unskilled labor by replacing $\widehat{\left(\frac{S}{K}\right)}$ with $\widehat{\left(\frac{U}{K}\right)}$ and found almost identical values (around 1.25) in both cases; this outcome is partly driven by the aforementioned identification problem. In another study, Oberfield and Raval (2014) used a nested-CES structure similar to Equation (1) with capital, labor, and materials (instead of two types of labor) and estimated $\sigma_{Agg}$ as a convex combination of the elasticity of demand, the elasticity of substitution between materials, and the capital-labor bundle. This provides a novel way of estimating $\sigma_{Agg}$, but the same identification problem persists since they combine capital and labor and effectively reduce the production structure from a three-input to a two-input case. Similarly, Elsby, Hobijn, and Sahin (2013) also contend that $\sigma_{Agg}$ becomes a weighted average of $\rho$ and $\sigma$[10]. Only Arpaia, Perez, and Pichelmann (2009) studied the differences in the elasticity of substitution between two kinds of labor (skilled and unskilled) and capital. However, none of these papers attempt to identify the differences between $\rho$ and $\sigma$ or to find their implications for the LIS trends.

In a multi-input nested CES production structure, the identification of both types of elasticities of substitution is crucial because they govern the links between changes in the relative factor income shares and changes in the relative factor prices (Blackorby and Russel, 1989; Anderson and Moroney, 1993). For example, using the nested-CES production function in Equation (1), the condition $\sigma > \rho$ implies that capital is more substitutable with unskilled labor than with skilled labor and the differential capital-skill substitutability can contribute to the LIS trend in various ways, including changes in the skill-premium through technological progress (discussed at length in KORV)[11]. In a similar vein, Diamond et al. (1978) cautioned that elasticities could only be identified when factor price movements are independent of the bias of technological changes.

In this paper, I propose an alternative framework to identify and estimate both types of elasticities of substitution ($\rho$ and $\sigma$) using a three-input nested-CES production structure. In a production structure with more than two inputs, the primary identification problem emerges from the simultaneous changes in the prices of factor inputs (other than the two directly used) to estimate elasticities of substitution. I use the concept of MES[12], which holds the prices of other factor inputs constant and adjusts the measure of the elasticity of substitution accordingly. Following the works of Blackorby and

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8 $L_S$ denotes the labor income share; $\widehat{L_S}$ denotes changes in the labor income share; $\hat{\delta}$ denotes changes in the relative price of investment goods (capital); $\widehat{\left(\frac{S}{K}\right)}$ denotes changes in the ratio between skilled labor and capital; $\alpha$ and $\beta$ are regression parameters (constants), and $\varepsilon$ denotes the idiosyncratic error term.

9 Karabarbounis and Neiman (2014) acknowledged this limitation in their paper (footnote 25).

10 Elsby, Hobijn, and Sahin (2013) demonstrate the relationship between labor income share ( $L_S$ ) and capital-skill complementarity as $dlnL_S = -(1-L_S)[\{\varphi\rho + (1-\varphi)\sigma\}dln\left(\frac{K}{L}\right) + \{\frac{S}{1-S}(1-\varphi)\rho - \varphi\sigma\}dln(S)]$.

11 KORV demonstrated capital-skill complementarity as the key feature of technology. They provided empirical evidence based on a theoretical framework that hypothesized elasticity of substitution was higher between capital and unskilled labor than between capital and skilled labor.

12 The Morishima elasticity of substitution is the natural multi-input generalization of the Hicksian two-input elasticity of substitution (Blackorby and Russel, 1989)

Russel (1989) and Anderson and Moroney (1993), I show that both $\rho$ and $\sigma$ can be approximated by the differences in own-price and cross-price elasticities in a nested-CES production framework. Moreover, MES directly links the changes in relative factor input prices to LIS trends. Using this framework with differential capital-skill substitutability ($\sigma \neq \rho$), I then derive the necessary and sufficient conditions for a decline in the LIS resulting from a drop in the relative price of capital. Assuming $\sigma > \rho$ (i.e., capital is more complementary to skilled labor), the necessary condition and the sufficient condition for a decline in the LIS with a drop in the relative price of capital are $\sigma > 1$ and $\rho > 1$, respectively.With capital-skill complementarity ($\sigma > \rho$), the sufficient condition $\rho > 1$ implies that $\sigma > \rho > 1$, which suggests that the aggregate substitution elasticity between capital and labor ($\sigma_{Agg}$) is greater than unity, thus supporting the findings of Karabarbounis and Neiman (2014). I derive conditions under which it is feasible to write the necessary condition $\sigma > 1$ as $\sigma > 1 > \rho$, which allows for one of the elasticities of substitution parameters to be less than unity, unlike the sufficient condition, $\sigma > \rho > 1$. With $\sigma > 1 > \rho$, it is feasible to have an estimate of $\sigma_{Agg}$ to be less than unity, which reconciles the contradictory outcomes related to the role of $\sigma_{Agg}$ behind the decline in the LIS.

The validity of the condition $\sigma > 1 > \rho$ remains an empirical question. This condition implies that we have gross complementarity between capital and skilled labor and gross substitutability between capital and unskilled labor; additionally, it indicates that a decline in the LIS with a drop in the relative price of capital occurs when the loss of LIS due to a decrease in the unskilled labor force outweighs the labor income gain due to an increase in the skilled labor force. Atkinson (2009) argued that the substitutability between capital and labor with heterogeneous labor provides a much richer set of possible distributional outcomes, and he suggested the use of MES to measure the degree of substitutability and complementarity between factors for a production function with more than two inputs. To the best of our knowledge, this is the first study that uses MES to identify the substitution parameters and that links them to explain movements in the LIS.

The economic rationale behind the use of capital-skill complementarity in explaining LIS trends comes from the well-documented trends showing an increase in the supply and relative wages of skilled labor over time. The literature on the skill-biased technical change (SBTC) argues that an increase in the demand for skills is a potential driver (Acemoglu and Autor, 2011) of this upward trend. The availability of cheaper capital equipment could also increase the demand for skilled labor with or without SBTC (Krusell, Ohanian, Rios-Lull, and Violante, 2000). More recently, Bueara, Kaboski, and Rogerson (2015) argued for a systematic reallocation of value-added shares toward high-skill intensive sectors, which they termed the skill-biased structural change (SBSC), to explain an increase in the supply of skills. This study is directly related to this literature on capital-skill complementarity.

This paper is also related to the growing literature on estimating $\sigma_{Agg}$ using disaggregated data (Oberfield and Raval, 2014; Herrendorf, Herrington, and Valentinyi, 2015). The literature on the estimation of σ is large [See Chirinko (2008) for a comprehensive summary], but plagued by subjective choices on parametric assumptions and functional forms of production (Leon-Ledesma, McAdam, and Willman, 2010). Oberfield and Raval (2014) used a novel micro-level framework to estimate $\sigma_{Agg}$; they considered both the elasticity of substitution between factor inputs within a plant and the reallocation of factor inputs across plants and found estimates of $\sigma_{Agg}$ to be less than one. Despite these insightful attempts, a consensus on the estimates of the aggregate elasticity of substitution between capital and labor is yet to be reached. This study provides an alternative framework to estimate $\sigma_{Agg}$ using disaggregated data.

Finally, this paper contributes to the growing literature on the drivers of the LIS. A large body of research has documented a global decline in the LIS and offers several explanations for this phenomenon (Elsby, Hobijn, and Sahin, 2013; Piketty, 2014). The assumption of a non-unitary elasticity of substitution (σ) between capital and labor plays a crucial role in explaining the changes in the LIS. The "accumulation view" assumes capital and labor to be gross substitutes ($\sigma_{Agg}$ > 1), whereas past studies have largely estimated $\sigma_{Agg}$ to be less than one (Oberfield and Raval, 2014; Chirinko and Mallick, 2017). To the best of our knowledge, this is the second paper to address this puzzle (after Grossman et al., 2017) and the first paper to provide an analytical framework to analyze movements in the LIS with $\sigma_{Agg}$ based on MES.

The rest of the paper is organized as follows. In Section 2, I provide a theoretical framework showing how MES can help to identify the elasticities of substitution in a production process with more than two inputs. Section 3 provides a comprehensive conclusion to the study.

# 2. THEORETICAL FRAMEWORK

I divide this section into two parts. First, I provide a brief introduction to MES and how it can be integrated into a CES production function. Second, I apply the MES concept directly to the changes in factor input prices and to the LIS trends through the channel of differential capital-skill substitutability.

## 2.1 MES in a Three-input Nested-CES Structure

A nested-CES production function (similar to equation 1) with three inputs suggests that the elasticities of substitution need to be different between the within-nest ($\rho$) and across-nest ($\sigma$) functions[13]. Hicks (1932) defined the elasticity of substitution as changes in the input use ratio resulting from changes in the marginal rate of technical substitution (MRTS) between inputs to analyze factor income shares. With two inputs $x_i$ and $x_j$, Hicks elasticity of substitution is $HES_{ij} = \frac{dlog\frac{x_i}{x_j}}{dlog(MRTS_{ji})}$. In a production function involving more than two inputs, the price of other inputs influences this substitution elasticity. MES addresses this issue and provides a natural generalization of the Hicksian two-input elasticity of substitution[14] (Blackorby and Russel, 1989).

As originally suggested by Pigou (1934), one way to address this issue is to hold output and other input factors, with the exception of one of the two in the ratio, constant. MES holds prices of other factor inputs constant and adjusts the measure of the elasticity of substitution accordingly. MES can be expressed as both a function of its own price and the cross-price elasticities of two inputs in the following way

[13] $\rho$ and $\sigma$ could be identical in special cases when the distribution parameters ($\theta$ and $\emptyset$) are identical and in the restricted CES structure that allows for MES to be symmetric (Blackorby and Russel, 1989).

[14] A number of alternative estimates have been developed (Hicks, 1934; Allen, 1938; Uzawa, 1962; McFadden, 1963; Morishima, 1967; Mundlak, 1968; Blackorby and Russel, 1989) to address such issues and to generalize the concept of elasticity of substitution for an arbitrary number of inputs ($i > 2$).

$$MES_{ij} = \frac{dlogx_j}{dlogp_i} - \frac{dlogx_i}{dlogp_i}{}^{15}. \quad (3)$$

It is evident from Equation (3) that $MES_{ij} \neq MES_{ji}$, i.e., MES is asymmetric as the value and the sign of MES differs between the price changes of input $x_i$ and changes in the price of input $x_j$. MES is biased towards gross substitution as the second term on the right-hand side of Equation (3) is always negative. For this reason, simple cross-price elasticities are preferred to MES if we intend to find out how the use of input 1 changes because of changes in the price of input 2 (Frondel and Schmidt, 2002). However, MES provides a direct link between the factor prices and the ratio of factor input uses. Blackorby and Russel (1989) showed that changes in the ratio of factor income shares can be directly predicted by MES using Equation (4):

$$\frac{dlog\frac{p_i x_i}{p_j x_j}}{dlog\frac{p_i}{p_j}} = 1 - MES_{ij}. \quad (4)$$

This property of MES makes it the right choice to study changes to LISs and the results of changes in relative factor prices.

In the next step, we derive similar expressions for MES using a three-input nested-CES structure. We rewrite the CES production structure in Equation (1) as a two-stage function consisting of two sub-processes, or nests, as follows:

$$Y = \left[\theta\left[\emptyset K^{\frac{\rho-1}{\rho}} + (1-\emptyset)S^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\frac{\sigma-1}{\sigma}} + \quad (1-\theta)U^{\frac{\sigma-1}{\sigma}} \quad \right]^{\frac{\sigma}{\sigma-1}} = N_1(K,S) + N_2(U). \quad (5)$$

From Equation (5), $\rho$ denotes the intra-nest elasticity of substitution between K, and S and $\sigma$ denote the inter-nest elasticity of substitution between K and U. The sub-processes $N_1$ (with inputs K and S) and $N_2$ (with just input U) are mutually exclusive and exhaustive. Following Anderson and Moroney (1993), who extended the work on MES for cases of more than two inputs, I write the expressions for $\rho$ and $\sigma$ as

$$\rho = MES_{KS} = \frac{dlogS}{dlogP_K} - \frac{dlogK}{dlogP_K}, \quad K, S \in N_1 \quad (6)$$

$$\sigma = MES_{KU} = \theta\left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] - \frac{dlogK}{dlogP_K}, \quad K \in N_1;\ U \in N_2. \quad (7)$$

In Equation (7), the expression $\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}$ refers to the MES between $N_1$ and $N_2$. Given the nested-CES production structure in equation 1, $\rho$ can be estimated through either $MES_{KS}$ or $MES_{SK}$ since an intra-nest MES is symmetric. However, if the inter-nest MES is asymmetric ($MES_{KU} \neq MES_{UK}$), the value of $\sigma$ will therefore change if we let the price of unskilled labor change instead of the price of capital (Blackorby and Russel, 1989; Anderson and Moroney, 1993). Assuming that at equilibrium factor prices equal the marginal product of each factor input, $P_K = r = \frac{dY}{dK}$=, $P_S = w_S = \frac{dY}{dS}$, and $P_U = w_U = \frac{dY}{dU}$, I write the expressions for both $\rho$ and $\sigma$ for two nested-CES production functions: $Y = f[(K,S)U]$ and $Y = f[(K,U)S]$ in Table 1. While the intra-nest MES ($\rho$) is simply the difference in the cross-price and own-price elasticities of the factor inputs,

[15] $p_i$ and $p_j$ are the prices of inputs $x_i$ and $x_j$.

for the inter-nest MES ($\sigma$) the own price elasticity factor is replaced by the MES across two nests ($N_1$ and $N_2$). Another point to note is that MES estimates vary across different nested-CES production functions and, particularly for inter-nest MES ($\sigma$), they vary subject to the relative prices of the factor inputs.

**Table 1: MES for Two Different Nested-CES Production Functions with Three Inputs**

| | (1) | (2) |
|---|---|---|
| | $Y = \left[\theta\left[\emptyset K^{\frac{\rho-1}{\rho}} + (1-\emptyset) S^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\frac{\sigma-1}{\sigma}} + (1-\theta) U^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = N_1(K,S) + N_2(U)$ | $Y = \left[\theta\left[\emptyset K^{\frac{\rho-1}{\rho}} + (1-\emptyset) U^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\frac{\sigma-1}{\sigma}} + (1-\theta) S^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = N_1(K,U) + N_2(S)$ |
| $\rho$ | $\rho = MES_{KS} = \frac{dlogS}{dlogr} - \frac{dlogK}{dlogr}$ <br> $\rho = MES_{SK} = \frac{dlogK}{dlogW_S} - \frac{dlogS}{dlogW_S}$ | $\rho = MES_{KU} = \frac{dlogU}{dlogr} - \frac{dlogK}{dlogr}$ <br> $\rho = MES_{UK} = \frac{dlogK}{dlogW_U} - \frac{dlogU}{dlogW_U}$ |
| $\sigma$ | $\sigma = MES_{KU} = \theta\left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] - \frac{dlogK}{dlogr}$ <br> $\sigma = MES_{UK} = \theta\left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] - \frac{dlogU}{dlogW_U}$ | $\sigma = MES_{KS} = \theta\left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] - \frac{dlogK}{dlogr}$ <br> $\sigma = MES_{SK} = \theta\left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] - \frac{dlogS}{dlogW_S}$ |

Note: At equilibrium, $P_K = r$, $P_S = w_S$ and $P_U = w_U$.
Source: Author.

## 2.2 An Application of MES to Study Movements in LIS

Using Equation (4) and the expressions for $\rho$ and $\sigma$ in Table 1, I propose a direct link between the changes in the relative prices of factor inputs and the LIS for the nested-CES production function $y = N_1(K,S) + N_2(U)$ as follows:

$$\frac{dlog\frac{W_S\,S}{r\,K}}{dlog\frac{W_S}{r}} = 1 - MES_{SK}(=\rho) = 1 - \frac{dlogK}{dlogW_S} + \frac{dlogS}{dlogW_S} \tag{8}$$

Equation (8) indicates that a drop in the relative price of capital (an increase in the ratio $\frac{W_S}{r}$) leads to a drop in the skilled LIS (i.e., $\frac{dlog\frac{W_S\,S}{r\,K}}{dlog\frac{W_S}{r}} < 0$) if $MES_{SK} > 1$. In other words, the skilled LIS declines due to cheaper capital when capital and skilled labor (intra-nest inputs) are gross substitutes. Similarly, Equation (9) shows that a drop in the relative price of capital (an increase in the ratio $\frac{W_U}{r}$) leads to a drop in the unskilled LIS (i.e., $\frac{dlog\frac{W_U\,U}{r\,K}}{dlog\frac{W_U}{r}} < 0$) if $MES_{UK} > 1$ or if capital and unskilled labor (inter-nest inputs) are gross substitutes.

$$\frac{dlog\frac{W_U\,U}{r\,K}}{dlog\frac{W_U}{r}} = 1 - MES_{UK}(=\sigma) = 1 - \theta\left[\frac{dlogN_2}{dlogP_{N_1}} - \frac{dlogN_1}{dlogP_{N_1}}\right] + \frac{dlogK}{dlogP_K} \tag{9}$$

From the MES expressions in Equations (6) and (7), it is likely that $\rho \neq \sigma$. If the elasticity of substitution between capital and skilled labor (intra-nest) is different from the elasticity of substitution between capital and unskilled labor (inter-nest), then the size of the gap between the values of $\rho$ and $\sigma$ has a direct bearing on the aggregate LIS changes. If both $\rho$ and $\sigma$ are greater than 1, i.e., capital is gross substitutes with both skilled and unskilled labor, then from Equations (8) and (9), it is sufficient to have a declining LIS with cheaper capital. I provide a more formal discussion of the necessary conditions in Proposition 1.

**Proposition 1: If $> \rho$, then (i) the necessary and (ii) sufficient conditions for a decline in the aggregate LIS with a drop in the relative price of capital are (i) $\sigma > 1$ and (ii) $\rho > 1$, respectively.**

Proof: I thus provide a rough sketch of the proof; at equilibrium (i.e., when marginal products equal factor prices) a simple expression for the LIS ($L_S$) can be written as

$$L_S = \frac{W_S\,S + W_U\,U}{y}, \text{ or } \frac{L_S}{1-L_S} = \frac{W_S\,S}{r\,K} + \frac{W_U\,U}{r\,K} \tag{10}$$

Taking log and differentiating Equation (10) with respect to the log of input-price ratios, we get

$$\frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{\overline{W}}{r})} = \frac{dlog\frac{W_S\,S}{r\,K}}{dlog\frac{W_S}{r}} + \frac{dlog\frac{W_U\,U}{r\,K}}{dlog\frac{W_U}{r}}. \tag{11}$$

If $L_S$ declines, then the sum of the signs of the terms on the right-hand side of Equation (11) must be negative. In Equation (11), $\overline{W}$ represents a representative (weighted) wage of the total labor market since $\sigma > \rho$, $\rho > 1$ ensures that both elasticities are greater than 1. Thus, it is clear from Equations (8) and (9) that the sign of the sum of the right-hand side terms of Equation (11) become negative when $\rho > 1$. Equation (11) is similar to the regression model (Equation 2) used by Karabarbounis and Neiman (2014), who found the substitution elasticities of capital for both skilled and unskilled labor to be greater than 1.

However, we do not need such strict conditions for a decline in the aggregate labor share. If $\sigma > \rho$ and $\sigma > 1$, then $\rho$ can be less than 1 as long as we have $\left|\frac{dlog\frac{W_S\,S}{r\,K}}{dlog\frac{W_S}{r}}\right| < \left|\frac{dlog\frac{W_U\,U}{r\,K}}{dlog\frac{W_U}{r}}\right|$ or $|1-\rho| < |1-\sigma|$. As is evident in Equation (11), in this case we can also have a decline in the LIS. This proves the required condition as $\sigma > 1$ and suggests that a decline in the labor share can also be obtained from a drop in the relative price of capital with a less strict condition when $\sigma > 1 > \rho$. Therefore, capital and unskilled labor must be gross substitutes for a decline in the LIS if the substitutability between capital and unskilled labor is higher than the substitutability between capital and skilled labor. For example, if $\sigma = 1.15$ and $\rho = .9$,[16] then $\frac{dlog\left(\frac{L_S}{1-L_S}\right)}{dlog(\frac{\overline{W}}{r})} < 0$ since $|1-.9| < |1-1.15|$.

[16] These parametric values are in line with the existing literature (Karabarbounis and Neiman, 2014; Rognile, 2015).

## 2.3 The Relationship between $\sigma_{Agg}$, $\sigma$, and $\rho$

As a final step, I show that it is feasible to have $\sigma_{Agg}$ < 1 under the condition $\sigma > 1 > \rho$. To proceed, we need to make some plausible assumptions about the relationship between $\sigma_{Agg}$, $\sigma$, and $\rho$. Oberfield and Raval (2014) derived a closed-form expression for $\sigma_{Agg}$, where the aggregate elasticity of substitution between capital and labor can be expressed as a weighted average of the sectoral (industry-level) elasticity of substitution parameters. Following Oberfield and Raval (2014), we can write

$$\sigma_{Agg} = (1 - \aleph)\varepsilon + \aleph\theta \tag{12}$$

where $\aleph$ represents a heterogeneity index, which takes a value of zero if the capital intensity is the same across sectors (or industries). $\varepsilon$ is the sectoral level elasticity of the substitution parameter and $\theta$ represents the elasticity of demand. Using a multisectoral model, Rognile (2015) derived an analytical solution to $\sigma_{Agg}$ as a function of five gross elasticities of substitution ($\sigma_Z, \sigma_F, \sigma_{G_1}, \sigma_{G_2}, \sigma_H$)[17]. He further highlighted the role of net elasticity of substitution (${\sigma_{Agg}}^{NET}$), which shows changes in the real capital to net output ratio and the net rental rate of capital. Following his "scarcity view," a decline in the LIS with a higher capital-output ratio can be attained using ${\sigma_{Agg}}^{NET} < 1$.

The primary goal here is to show the existence of a set with feasible values for $\sigma$ and $\rho$, for which $\sigma_{Agg}$ < 1 under the condition $\sigma > 1 > \rho$. Following the current literature on the elasticity of substitution and to keep the task tractable, in Equation (13) I write the aggregate elasticity of substitution between capital and labor as a weighted average of $\sigma$ and $\rho$ (with $y$ and $x$ as weights)

$$\sigma_{Agg} = y\sigma + x\rho. \tag{13}$$

Imposing the condition for aggregate complementarity between capital and labor, Equation (13) becomes an inequality; after applying the previous numerical example with $\sigma = 1.15$ and $\rho = .9$, it becomes

$$1 > 1.15y + .9x \text{ or } y < .869 - .78x. \tag{14}$$

The feasible range of values that satisfy Equation (14) are plotted in Figure 1. Any point (combination of weights) in the shaded area implies that the weighted average of $\sigma$ and $\rho$ (with $y$ and $x$ as weights) must be less than the unity for the given values of $\sigma = 1.15$ and $\rho = .9$ . This hypothetical example suggests that it is possible to have complementarity between capital and labor ($\sigma_{Agg}$ < 1) for a feasible set of values of $\sigma$ and $\rho$, which corresponds with a decline in the LIS.

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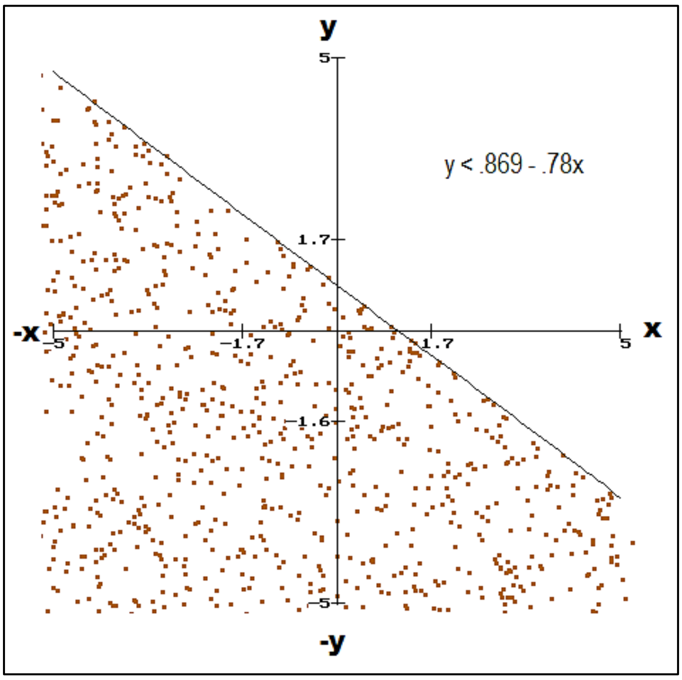
[17] $\sigma_Z$ is the elasticity of demand for housing services; $\sigma_F$ is the elasticity of substitution between real estate and other services; $\sigma_{G_1}$ is the elasticity of substitution between structures and land in non-housing sectors; $\sigma_{G_2}$ is the elasticity of substitution between structures and land in the housing sector; and finally, $\sigma_H$ represents the elasticity of substitution between equipment and labor.

**Figure 1: A Feasible Range of Weights for $\sigma_{Agg}$ < 1**




Note: This graph shows a numerical example. It shows the feasible range of value for an equation showing inequality.

Source: Author.

# 3. CONCLUDING REMARKS

Economists have always been concerned with the functional distribution of income. David Ricardo's famous statement, published back in 1817, serves as a testimony to this fact, "*To determine the laws which regulate [this] distribution is the principal problem in political economy*" Ricardo (1911, 1, 2nd ed). As emphasized by both Atkinson (2009) and Glyn (2009), the study of factor income shares plays an important role in understanding the relationship between national income and personal income, the relationship between wage inequality and wealth inequality, and how they link to overall income inequality and concerns for fairness in different sources of income. In recent years, a large body of research has documented a global decline in the LIS. The downward trend of the LIS has important implications for economic growth and income distribution. Moreover, a recent study by Kanbur, Rhee, and Zhuang (2014) noted that, since the 1990s, the declining LIS has contributed to rising income inequality in many Asian countries.

The burgeoning literature on the LIS offers several explanations for its global decline (Elsby, Hobijn, and Sahin, 2013; Piketty, 2014). According to the "accumulation view", a decline in the LIS is either driven by a decline in the relative price of capital (Karabarbounis and Neiman, 2014) or an increase in the capital-income ratio (Piketty, 2014). In the presence of heterogenous labor with capital-skill complementarity (Krusell et al., 2000), the sufficient condition for the "accumulation view" is $\sigma > \rho > 1$, where $\sigma$ and $\rho$ stand for the elasticity of substitution between capital and unskilled labor and capital and skilled labor, respectively (Arpaia, Perez, and Pichelmann, 2009; Karabarbounis and Neiman, 2014). The condition $\sigma > \rho > 1$ implies that the aggregate substitution elasticity between capital and labor ($\sigma_{Agg}$) is greater than unity, which extant literature predominantly estimates $\sigma_{Agg}$ to be less than one (Oberfield and Raval, 2014; Chirinko and Mallick, 2017).

This paper uses the MES to identify $\sigma$ and $\rho$. With capital-skill complementarity ($\sigma > \rho$), an application of MES provides the necessary conditions for a decline in the LIS resulting from a decrease in the relative price of capital as $\sigma > 1 > \rho$, which is more flexible than the sufficient condition. With $\sigma > 1 > \rho$, it is feasible to estimate $\sigma_{Agg}$ to be less than unity, which reconciles the contradictory outcomes related to the role of $\sigma_{Agg}$ in the decline of the LIS. This condition tends to imply that a decline of the LIS alongside a drop in the relative price of capital occurs when the loss of income share due to a decrease in the unskilled labor force outweighs the income gained due to an increase in the skilled labor force.

The necessary and sufficient conditions refine our understanding of the drivers of a declining LIS. It is possible that in the presence of a segmented labor market, a decline in the aggregate LIS can be driven by one segment of the labor market. The relevance of capital-skill substitutability in studying changes in the labor share of income can also be drawn using a two-stage production structure (Goldin and Katz, 1996). In the first stage, skilled workers adopt new technologies and efficiently use capital, thus showing high capital-skill complementarity. In the second stage, unskilled workers continue the mechanical process of machine maintenance indicating a relatively low level of capital-skill complementarity. Such practices are common across both developing and developed countries and provide an important link between capital-skill substitutability and factor income shares.

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