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Money Changes Everything: Funding Shocks and Optimal Admissions and Financial Aid Policies in Higher Education

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The paper examines the effect of a shock to university funding on tuition net of financial aid, admissions selectivity, and enrollment levels chosen by an optimizing university. Whereas a positive shock, such as a major donation, results in lower net tuition and greater selectivity with respect to all students, its effect on enrollment may not be uniform. Student categories given little weight in the university's utility function may be treated as "inferior goods," that is, their enrollment may be decreased in the face of a positive shock, while other student categories see enrollments increased. Such students are charged net tuition well above their marginal cost of enrollment and play primarily a revenue- rather than prestige-generating role for the university. Inferences are drawn from the analytical framework concerning the effect on tuition levels of an increase in federal direct-to-student aid, permitting a new perspective on evidence relating to the Bennett hypothesis.

Keywords

financial aid policy, admissions policy, educational finance, utility maximization models

Comments

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PRELIMINARY DRAFT – DO NOT CITE WITHOUT PERMISSION

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1. Introduction

On August 1, 2006, Stanford University announced that Philip H. Knight, founder and chairman of Nike, Inc., would give $105 million to its Graduate School of Business ("GSB"). The gift is believed to be the largest ever made to a business school.[1] The event provides a stark example of a funding shock – a major change, whether positive or negative, in the level of funding available from endowment and external sources for university expenditures. Other examples include receiving a major research grant, experiencing a sharp drop in the value of endowment investments, or experiencing a cut in state appropriations.

A funding shock may be a highly significant event for an institution of higher learning. The endowment of Stanford GSB was valued at $711.8 in 2005, so Knight's donation increased it by about 15%.[2] Events of this magnitude may affect financial aid, admissions, and enrollment decisions in important ways.[3] Given the recent prevalence of both record donations to universities and colleges and ongoing cutbacks in federal and state funding, seeking an improved understanding of the impacts of funding shocks would seem warranted.

This paper employs a theoretical model to examine the effect of a funding shock on optimal university decision-making with respect to the level of tuition charged to students, net of financial aid ("net tuition"); admissions selectivity; and enrollment. The model considers a situation in which different student groups are characterized by different levels of

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[1] See "Record Donation for Stanford," *The San Jose Mercury News*, accessed on August 6, 2006, at http://www.mercurynews.com/mld/mercurynews/living/education/15178693.htm.

[2] Stanford University's endowment is $15 billion; the donation increases this composite figure by less than 1%. But because professional schools are often treated as autonomous units financially, the 15% figure is probably more relevant.

[3] Though funding inflows often are specifically earmarked – $100 million of Knight's gift, for example, will be devoted to constructing a new campus for the business school – they free up other university funds for general use.

relative attractiveness to the university, independent of objectively measurable quality.[4] Enrolled students with greater relative attractiveness carry greater weight in the university's utility function; thus the university cares more both about enrolling more attractive students and about their relative quality when enrolled.

A key recognition of the model is that students whose net tuition is set high relative to their costs of attending, whether because their relative attractiveness to the university is low or because the university's resources are limited, are treated differently from other students by an optimizing university when a funding shock occurs. I find that a positive shock, such as a major donation, lowers net tuition to all students, a result consistent with existing empirical work (Lowry, 2001; Rizzo & Ehrenberg, 2004). Not surprisingly, it also increases the university's admissions selectivity with respect to all students, as the university balances competing goals of higher quantity and quality of students. However, a positive funding shock results in *lower* enrollment for students whose net tuition is sufficiently high relative to cost, while resulting in *higher* enrollment for all others. Conversely, a negative shock results in higher net tuition and lower selectivity with respect to all students, but increased enrollment for high-margin students, whereas other students' enrollments are cut. For this reason, I refer to high-margin students as "inferior goods" to the university, as the effect of additional funding on the university's enrollment of these students is analogous to the effect of additional income on the quantity an individual consumes of an inferior good.

There has been some recognition in the literature that certain student groups may be singled out by universities for use as revenue generators. Mixon & Hsing (1994) state that public university administrators and state government officials take a particular interest in out-of-state students as a source of revenue, given that they typically pay a higher tuition

[4] I follow Ehrenberg & Sherman (1984) in making this assumption.

price. The authors go so far as to point out the importance to states of in-migration of students in light of recent budget cuts in higher education. Balderston (1997) offers anecdotal evidence from the University of Michigan of the use of out-of-state enrollment to increase revenue. However, it does not appear that anyone has systematically studied under what conditions a student group will be targeted in this way.

In addition to shedding light on the effect of funding shocks *per se*, the model offers a comparative static framework that can be extended to analyze the effects on net tuition, selectivity, and enrollment of a wide array of changes in market conditions. To illustrate, I use the model to analyze the effect on net tuition of a shock to direct-to-student financial aid. Some policymakers and analysts have contended that federal direct-to-student aid creates incentives for universities to raise their tuition levels. This so-called "Bennett hypothesis" has received considerable scholarly attention, with a number of studies investigating its validity (e.g., McPherson & Schapiro, 1991; Hauptman & Krop, 1998; Turner, 1998; Cunningham, Wellman, Clinedinst & Merisotis, 2001).

The rest of the paper is structured as follows. Section 2 briefly reviews the related literature. Section 3 describes the model. In Section 4, the main results relating to the effects of funding shocks are presented. In Section 5, the effect of an increase in federal direct-to-student aid is considered. Section 6 concludes.

2. Related literature

There is a substantial theoretical literature that uses utility-maximization models to describe university behavior. Garvin (1980) and James (1990) offer detailed models in which university utility depends on the quantity and quality of students and faculty (or, equivalently, research). Both analyses focus primarily on specifying the university's problem correctly and examining first-order conditions. Garvin discusses a few very general comparative static results to exemplify the uses of his model. Using a framework more closely resembling that in the present paper, Ehrenberg & Sherman (1984) focus on the student side of the university's problem, abstracting from issues relating to faculty and administrative inputs. They consider how optimal financial aid packages should be set, given varying student demand characteristics and basic variations in relative university preferences across student groups. They draw a number of important conclusions about financial aid decision making, but do not address how changes in funding, as reflected in the university's budget constraint, should affect such decisions.

Two other theoretical analyses closely related to the present paper are offered by Fethke (2005, 2006), who uses a principal-agent framework to study the relationship of state appropriations to public university tuition levels in a context in which states compete. In a decentralized baseline case, the state legislature chooses a higher education subsidy level to maximize a welfare function that sums the consumer surplus of students, university revenue, and direct utility from university enrollment.[5] Meanwhile, the university cares only about its revenue and student surplus; it does not, independently, care about enrollment.

Fethke (2005) performs a positive economic analysis on this framework. He determines that the subsidy and tuition levels chosen depend upon the university's and legislature's relative preferences for revenue versus student surplus. The paper also provides results regarding how relative preference levels for student surplus affect how subsidy and

[5] In Fethke (2006), the legislature cares about enrollment only for resident (in-state) students; consequently, as described below, this paper distinguishes differential effects on resident and non-resident students of changes in various conditions.

tuition respond to increases in the demand for and marginal cost of education. Fethke (2006) focuses on normative considerations, deriving vertical coordination strategies for arriving at optimal subsidy and tuition levels. However, the paper also derives two descriptive conclusions that extend across the alternative strategic arrangements considered. First, the difference between non-resident and resident tuition declines with decreases in state appropriations. Second, it declines with increases in the demand for or cost of education.

An important characteristic of Fethke's framework is that he assumes state legislatures set subsidies strategically, recognizing the effect that they have on university tuition-setting. Thus, he endogenizes the subsidies in the tuition model. In contrast, I focus on the effects of exogenous shocks to university funding; this means, to the extent that my model applies to state appropriations, it assumes them to be exogenously determined.

Also relevant to the present paper are a number of empirical studies that have considered the effects of state appropriations on tuition and enrollment at public institutions of higher education. Estimating a simultaneous equations system using cross-sectional data, Koshal & Koshal (2000) find that lower state appropriations for higher education at in-state public universities are associated with higher tuition. Similar results are found by Cunningham et al. (2001) for public research/doctoral and public comprehensive institutions. Lowry (2001) estimates the determinants of the major components of university revenue, including state appropriations and tuition and fees. He finds that tuition and fee revenues are higher at institutions that receive less state funding per student. Estimating a simultaneous equations system on panel data, Rizzo & Ehrenberg (2004) find that higher state budget appropriations correlate with lower in-state and out-of-state tuitions at public universities. They also find using a cross-section approach that state appropriations correlate negatively with out-of-state share of enrollment. This outcome seems consistent with the idea that out-of-state students are tapped to replace revenue lost to state funding cuts; however, the authors do not find that the result holds when a difference-on-difference panel approach is applied using the same data. Collectively, these studies of state appropriations offer insight into some of the ramifications of funding shocks, by reference to an important example of such shocks. But no work has yet provided a conceptual understanding of how funding shocks, as a general phenomenon, affect key university decisions and, particularly, how they differentially affect different groups of students.

3. Model

Consider a university enrolling students from two different groups, $i = 1, 2$. The groups may be thought of as consisting of athletes and non-athletes, state residents and non-resident students, poor and wealthy students, or any other appropriate dichotomy. The university charges "net tuition," P_i, to group *i*, where net tuition consists of a positive level of full tuition, minus a nonnegative amount of institutional financial aid. Net tuition may be viewed as differing for the two groups either because full tuition differs, financial aid packages differ, or both.[6]

The net tuition level P_i generates a demand for admission (i.e., number of applicants), $Q_i\left(P_i\right)$. The groups' demands are mutually independent and are given by a general functional form (Genesove & Mullin, 1998),

[6] Under certain circumstances, universities may set different full tuition levels to different students, such as resident and non-resident students. For a discussion of tuition differentials, see Balderston (1997).

$$Q_i(P_i) = b_i(a_i - P_i)^{\gamma_i} \tag{1}$$

where $a_i > 0$ is the student's maximum willingness to pay, $b_i > 0$ represents the size of the market, and $\gamma_i > 0$ is an index of convexity. Note that linear ($\gamma_i = 1$) and quadratic ($\gamma_i = 2$) demand curves, among other forms, are nested as special cases. The university is generally considered to be selective, that is, it does not enroll every student who applies. Rather, it accepts a share $0 \le \phi_i \le 1$ of the applicants from group *i*. I assume that all accepted students enroll at the university, so enrollment E_i is defined $E_i \equiv \phi_i Q_i$.

The university obtains utility from enrolling students. Its values are summarized by a weighted-enrollment utility function,

$$U = \alpha h_1(\phi_1) E_1(\phi_1, P_1) + (1-\alpha) h_2(\phi_2) E_2(\phi_2, P_2) \tag{2}$$

Here, $h_i(\phi_i)$ is the *average quality level* of accepted applicants in group *i,* and the weight $0 \le \alpha \le 1$ reflects the *relative attractiveness* of the two groups to university. One may think of h_i as reflecting a composite of objective measures of academic quality, such as test scores and grade-point average. Meanwhile, α reflects the level of subjective preference on the part of university administrators for group 1 relative to group 2.[7] Let h_i be defined as a linear function of ϕ_i,

$$h_i(\phi_i) = \mu_i + \sigma_i \phi_i \tag{3}$$

[7] For a more extensive discussion of objective academic quality versus subjective attractiveness, see Ehrenberg & Sherman (1984).

where $\mu_i > 0$ and $0 > \sigma_i > -\mu_i / \phi_i$. Thus, h_i is a decreasing function of the share of applicants accepted, and is positive and bounded on its domain.[8]

The university's objective is to choose net tuition levels P_i and acceptance shares (or "selectivity" levels) ϕ_i to maximize (2) subject to breaking even on its operating account. This break-even requirement is summarized by the following resource constraint

$$R \equiv (c_1 - P_1) E_1(\phi_1, P_1) + (c_2 - P_2) E_2(\phi_2, P_2) - F \leq 0 \tag{4}$$

where $c_i > 0$ represents the marginal cost of enrolling students from group *i*, and F represents funds available from the endowment, government appropriations, and other non-student sources. In words, (4) states that the net cost of enrolling students (after net tuition is subtracted out) must not exceed the funds available from non-student sources.

In the context of this framework, a change in F will be termed a *funding shock*. Note that a positive funding shock is equivalent to a relaxation of the resource constraint, while a negative shock equates to a tightening of the constraint. As discussed in the introduction, the primary purpose of the analysis will be to understand how funding shocks affect the net tuition, selectivity, and enrollment decisions of the optimizing university. Differentiating the expression for enrollment with respect to F and evaluating at the optimizing (ϕ_i^*, P_i^*) pair gives:

$$\frac{\partial E_i}{\partial F} = \phi_i^* \frac{\partial Q_i}{\partial P_i} \frac{\partial P_i^*}{\partial F} + Q_i \frac{\partial \phi_i^*}{\partial F} \tag{5}$$

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[8] Unlike Ehrenberg & Sherman (1984), I do not impose the restriction that total quality units never decrease with the number of students admitted, i.e., $\phi h_i'(\phi_i)/h_i > -1$. The assumption is not necessary to ensure positive enrollment in the present model. Further, by not imposing this restriction, one allows for the possibility that admitting low quality students might decrease the utility of the university, perhaps by damaging its prestige.

When the expression in (5) is positive, a positive shock to funding increases the equilibrium enrollment of group *i*. This is intuitively what one would expect to happen to enrollment when more money is available to the university; after all, (2) shows that the university obtains utility from enrolling students. But when this expression is negative, a positive shock to funding decreases the enrollment of group *i*. This would seem to represent an anomaly. Why would a university cut back the number of students it enrolls from a group when it has more money with which to subsidize enrollment? Paralleling the terminology of consumer theory, I will refer to a group of students *i* for whom $\frac{\partial E_i}{\partial F} > 0$ as "normal goods" to the university. A group of students for whom $\frac{\partial E_i}{\partial F} < 0$ will be referred to as "inferior goods." The main focus of the next section will be to explain when and why the anomalous outcome – inferiority – occurs.

4. Results

4.1 Non-selective case

Let us begin by considering the special case in which the university is not selective, that is, ϕ_i is fixed at 1. Note that the second term of (5) vanishes in this case because ϕ_i is fixed. Given $\frac{\partial Q_i}{\partial P_i} < 0$, the sign of $\frac{\partial E_i}{\partial F}$ depends solely on the sign of the effect of a funding shock on net tuition, $\frac{\partial P_i^*}{\partial F}$, which may be derived using comparative static techniques. The following result is obtained (all propositions, lemmas, and corollaries are proved in the appendix):[9]

PROPOSITION 1: At a non-selective university, negative funding shocks should result in higher net tuition for all groups and reduced enrollment for all groups. Therefore, no groups are inferior goods.

A non-selective university has only one instrument with which to both manage enrollment and collect revenue from each group – net tuition. When funding is cut, it must use this instrument to raise the needed additional revenue. If lowering tuition to any group would have increased revenue from that group, the university would already have done this before funding was cut, because doing so would also have increased enrollment. Therefore the effects of a funding cut on net tuition and enrollment are unambiguous.

4.2 Selective case

Now assume the university can both set net tuition and decide on a portion of applicants to admit from each group of students. The sign of $\frac{\partial E_i}{\partial F}$ now depends on the signs of $\frac{\partial P_i^*}{\partial F}$ and $\frac{\partial \phi_i^*}{\partial F}$, as well as the relative size of the two terms in (5). Again, comparative static techniques are used to make the necessary determinations. First, it may be shown that:

[9] This proposition and all other model results pertaining to the effects of funding shocks are written in terms of the effect of a *negative* shock (i.e., a *reduction* in funding). The effect of positive funding shock is simply the inverse of what is stated in each case.

PROPOSITION 2: At a selective university, negative funding shocks should result in higher net tuition for all groups and reduced admissions selectivity for all groups.

By lowering standards, the university is able to admit more students, thus offsetting somewhat the reduction in enrollment that occurs from raising net tuition. Though doing so means the university must accept some reduction in average quality, it optimally balances this cost against the cost of reducing enrollment. The new balance that is struck following a reduction in funding involves some increase in net tuition and some reduction in selectivity.

Before turning to the conditions for inferiority in the selective case, it is helpful to understand the determinants of the sign of the tuition-cost margin, $P_i - c_i$. These are laid out in the following proposition:

PROPOSITION 3: Students should be charged net tuition greater than their marginal cost of enrollment if their group's relative attractiveness to the university is low enough or if endowment and other non-tuition funding sources are sufficiently limited.

The proposition formalizes the conditions that result in a group of students cross-subsidizing other activities of the university.[10] Not surprisingly, if a group is of very low relative attractiveness to the university, its tuition revenues will be used to fund the enrollment of more attractive groups. But a student group may end up subsidizing university operations even if it does not exhibit low relative attractiveness, if the university is truly impoverished.

[10] See James (1990).

The following proposition establishes an important qualitative result relating to the $P_i > c_i$ case:

PROPOSITION 4: When a group is charged net tuition in excess of its marginal cost of enrollment, the university admits from the group some students that are of such low quality that they make a negative contribution to the university's utility.

The university admits students from a group until the utility contributed by the marginal student just equals the net monetary cost of enrolling that student. Since $P_i > c_i$, the university incurs a *negative* net cost (i.e., earns a positive return) from enrolling students. This means that the marginal student will provide the university with negative utility.

One way to think about the meaning of this is to think of a university's student body as consisting of two types of students: prestige generators and revenue generators. On the one hand, a selective university desires students that reflect well on the institution, and it will often be willing to pay handsomely to have such students, offering them lavish financial aid packages. On the other hand, the university may be willing to take on some real "undesirables" from a prestige perspective, if it gets sufficient money to do so. So, though we might think of the former type of student as "favored" and the latter "disfavored" by the university, it is perhaps truer to say that the university wants both types of student around, but for different reasons.

Now let us return to the question of inferiority. The following proposition offers the main result with respect to the effect of a funding shock on equilibrium enrollment:

PROPOSITION 5 (Inferiority condition): Negative funding shocks reduce the equilibrium enrollment level for all groups, except for groups for whom $\frac{P_i - c_i}{P_i} > \frac{\gamma_i}{\gamma_i + 1}\frac{1}{\varepsilon_i}$, where ε_i is group *i*'s demand elasticity. For these groups, negative shocks should result in an increase in enrollment.

That is, when a group's tuition-cost margin is sufficiently elevated given demand elasticity (and the extent of demand convexity), funding cuts should actually cause the group's enrollment to increase. In effect, when more money is needed, the university manipulates net tuition and selectivity to increase the enrollment of groups whose members generate sufficient revenue. Note that the higher the demand elasticity, the lower the threshold for inferiority (i.e., closer to $P_i = c_i$). Intuitively, a university will raise net tuition less in the face of funding cuts for groups exhibiting greater demand elasticity, making it more likely that their enrollment will increase rather than decrease.

This finding leads to an important general prediction: the optimizing university's student population will shift in favor of student groups that cover their costs when money is tight. Conversely, it will generally shift in favor of groups that depend on subsidization when money is easy. Put another way, the composition of a student body can be expected to shift with the university's financial fortunes, alternating between prestige-generating students and revenue-generating students.

While Proposition 5 has intuitive appeal in that it makes a connection between inferior good status and a group's tuition-cost margin, it would also be useful to connect inferiority with exogenous group characteristics. Hence, the following corollary is offered:[11]

COROLLARY 1: An optimizing university will increase a group's enrollment in the face of a negative funding shock if the group exhibits sufficiently low relative attractiveness to the university, or if endowment and other non-tuition funding sources are sufficiently limited.

Note that if a university's resources are sufficiently constrained, all its students could be inferior goods. Funding cuts would lead to increased enrollment for all, while increases in non-tuition funding would lead to a general reduction in enrollment. This may seem topsy-turvy, but recall that when a group is charged net tuition greater than its marginal cost of enrollment, some students are enrolled who make a negative contribution to the university's utility. Reducing enrollment for such groups by increasing admissions standards actually increases utility. Thus, "weeding out" undesirable students from a body of applicants may be thought of as a luxury that the resource-constrained university enjoys when it receives a positive shock to its funding.[12]

A stylized fact that follows from Proposition 2 is that, for normal good students, the enrollment effect of changes in net tuition brought about by funding shocks dominates the

[11] The result parallels Proposition 3.

[12] The inferior good status described in the model is more properly termed "quasi-inferiority," because the university still enrolls more *quality units* from the inferior good group when a positive funding shock occurs. For true inferiority, it would be necessary for the marginal utility to the university of a group of students to be dependent on the number of students enrolled from the other group. Thus, the university would enroll fewer inferior good students because it is substituting normal good students and no longer needs the former. Since I have made the utility function additively separable, however, the marginal utilities are independent. Note that the phenomenon of quasi-inferiority is what allows for the possibility that all student groups are "inferior goods." With true inferiority, an economy must always have at least one normal good.

enrollment effect of changes in selectivity, while for inferior good students, the opposite is true. Thus:

PROPOSITION 6: Given equivalent demand for admission, Q_i, and equivalent responsiveness of enrollment to changes in net tuition, $\phi_i \frac{\partial Q_i}{\partial P_i}$, for both types of groups, normal good student groups experience greater volatility in net tuition and lower volatility in admissions standards in response to funding shocks than inferior good students.

All else being equal, inferior good students' average quality will tend to be highly variable in response to funding shocks. Meanwhile, these students will not tend to suffer large fluctuations in their cost of attending as a consequence of such shocks. However, normal good students will suffer such fluctuations, while their average quality level will tend to be more stable.

5. Effect of an increase in federal direct-to-student aid

In this section, I use the framework introduced above to investigate a controversial issue raised with respect to federally administered financial aid. William Bennett, the Secretary of Education under President Reagan, asserted in the mid-1980s that federal financial aid programs "enable" universities and colleges to increase their tuition by allowing them to rely on aid to cushion the effects of increases (Bennett, 1987). Analysts have provided two main rationalizations of Bennett's assertion.[13] First, properly-targeted aid shifts outward the demand curve for higher education by increasing the number of individuals that can afford to attend. This, it is argued, motivates institutions to increase tuition by making such increases more profitable. Second, federal aid programs create direct incentives for raising tuition by making aid awards dependent upon student need, where "need" is defined in part based on the size of tuition.

To this day, Bennett's hypothesis remains a topic of intense debate. A number of studies have attempted to discern whether federal direct aid leads to increased tuition, and if so, under what circumstances this occurs. Hauptman & Krop (1998) point to trend data as evidence that the increased availability of federally guaranteed loans has accommodated increases in tuition. McPherson & Schapiro (1991) analyze a simultaneous equations model incorporating number of university decision variables. They find no evidence of a relationship between federal grant aid and tuition increases for private institutions. However, they do find a significant positive relationship for public 4-year institutions. This they attribute to the fact that public institutions might have tuition low enough that raising it could affect student qualification for major federal award programs, such as the Pell grant or Guaranteed Student Loan. Turner (1998) turns the issue on its head, asking to what extent the benefits of the Pell grant program flow to its intended beneficiaries. Her theoretical analysis suggests that universities already giving substantial institutional aid to needy students may seize on Pell grants as an opportunity to free up those funds for other uses, thus partially diverting Pell program benefits. Cunningham et al. (2001) perform a series of reduced-form regressions to explain changes in tuition at seven different types of higher education institutions (e.g., public research/doctoral, public comprehensive, etc.). They find

[13] See, e.g., McPherson & Schapiro (1991), p. 68.

no relationship between any form of external grant aid, including federal grant aid, and changes in tuition at any of the seven institution types.

In the wake of this work, one conceptual issue that remains open is whether the increase in demand brought about by a positive shock to federal direct-to-student aid would indeed lead to an unambiguous increase in tuition. In the model introduced in Section 3, a shift in demand may be represented by a change in b_i in (1). It can be shown that the effect of an increase in demand on net tuition is proportional to the effect of a positive funding shock on net tuition, where the proportionality factor is[14]

$$-\left(c_i - P_i\right)\phi_i\left(a_i - P_i\right)^{\gamma_i} \tag{6}$$

Proposition 2 establishes that a positive funding shock results in decreased net tuition, so the effect of an increase in demand takes the sign of $c_i - P_i$. In words, an increase in demand for education results in higher net tuition when net tuition is less than marginal cost, and lower net tuition when net tuition is greater than marginal cost. Intuitively, an increase in demand causes the university to lower net tuition if it generates revenue for the university (by increasing the enrollment of revenue-generating students), while it causes the university to raise net tuition if it drains revenue away (by increasing the enrollment of subsidized, prestige-generating students).[15]

Consider the implications of this for the Bennett hypothesis. First, an increase in demand does not necessarily lead to increased tuition at a utility-maximizing university the way it would at a profit-maximizing firm. This suggests that Bennett may have based his hypothesis, at least in part, on an incorrect assumption. Second, caution should be exercised

[14] See appendix.
[15] This appears consistent with Fethke's (2006) result that the gap between nonresident and resident tuition at public universities varies negatively with the (overall) demand for higher education

in interpreting McPherson & Schapiro's (1991) differing results for public and private institutions as indicating that public universities are trying to game the structure of the Pell program. Given a mission to provide an affordable education to resident students, most public universities probably have a greater percentage of students paying net tuition below marginal cost than private universities. Thus, public four-year institutions might see average tuition levels increase with increases in federal student aid because the average student pays net tuition below cost. Meanwhile, private four-year institutions might not see an increase in their average tuition level because the average student pays net tuition close to or above cost. McPherson & Schapiro's (1991) results may simply reflect differences in the tuition-cost margins of students at public versus private institutions.

6. Conclusion

At an inn visited by the author while researching this paper hangs a sign that reads, "All visitors bring us happiness – some when they come, and some when they leave." In perhaps a similar spirit, as this paper has suggested, all students benefit the university – some by bringing prestige or an uplifting sense of having served some community of interest, and some by bringing money. The model developed in the paper has demonstrated that, in the face of a negative shock to university funding, the former group's enrollment is reduced, while the latter group's enrollment is increased. The opposite occurs when there is a positive shock to funding. The university's "favored" students (those bringing prestige) experience greater volatility in net tuition than the "disfavored" students (those bringing money), while the disfavored students experience greater volatility in admissions standards than the favored students. This makes it possible for favored and disfavored student enrollments to be negatively correlated in the face of a funding shock, while both groups of students experience the same direction of movement of net tuition and admissions selectivity.

As the model has demonstrated, in instances where university resources are extremely tight, students may be charged high net tuition relative to cost even when they exhibit high relative attractiveness to the university. Thus, even student groups that are relatively favored may play primarily a funding role for the university, so their enrollment levels may vary negatively with funding shocks.

The model's results were applied to analyzing the question of whether increases to federal direct-to-student aid lead universities to increase tuition. It was found that, for a utility-maximizing institution, increases in tuition do not necessarily follow from increases in demand. The particular pattern of increases predicted by the model is consistent with empirical outcomes previously attributed to public universities strategically increasing tuition to take advantage of the structure of federal aid programs. The model shows, in effect, that this pattern may result without such strategic behavior.

The results have important implications. James (1990) states that public universities face greater uncertainty with respect to their funding than private universities, because they are dependent for that much of that funding on the whim of a small group of people – state legislators and executives. It seems obvious, therefore, that public universities will face greater vicissitudes of tuition, admissions standards, and enrollment. But the model suggests something more subtle, that there are two constituencies affected differently by the uncertainty of life at the public university. There are subsidized students whose cost of attending fluctuates substantially; and subsidizing students, of whom the average academic quality level fluctuates substantially (to the extent that they are “inferior goods”). Public universities may anticipate unique social problems and challenges for their student communities as a result of this mix.

There are other implications for public university administrators. It has been suggested that governing boards should raise nonresident tuition to offset losses of university revenue that follow from decreases in state appropriations. The model in this paper suggests that, when nonresidents are being priced at a large margin already, the key is rather to manipulate tuition and admissions standards together to increase nonresident enrollment. Though this may seem undesirable to some public universities, it appears to be the most effective way to avoid substantially raising the tuition or curbing the enrollment of residents.

The predictions of the model should be examined empirically. An empirical analysis that considers common effects across a range of funding shocks would contribute to our understanding of these phenomena. It would also be useful to re-examine the effects of changes in state appropriation levels, taking account of the possibility that different students may be affected by these changes in different ways. Finally, the effect of federal student aid on tuition should be re-examined, accounting for different impacts on different groups of students.

Appendix A. Proofs

Proposition 1: The non-selective university’s problem is to choose P_i to maximize (2) subject to (4) and $\phi_i = 1$. The corresponding Lagrangian function is

$$
\begin{aligned}
L_{NS}\left(P_1, P_2, \lambda\right) &\equiv \alpha \underline{h}_1 Q_1\left(P_1\right) + \left(1-\alpha\right)\underline{h}_2 Q_2\left(P_2\right) \\
&+\lambda\left[\left(c_1 - P_1\right)Q_1(P_1) + \left(c_2 - P_2\right)Q_2(P_2) - F\right]
\end{aligned}
\tag{A1}
$$

where $\lambda < 0$ and $\underline{h}_i \equiv \mu_i + \sigma_i > 0$. Let us assume an interior solution, so that the constraint in (4) is binding. The first-order conditions ("FOCs") are

$$
\begin{aligned}
&\alpha \underline{h}_1 \frac{\partial Q_1}{\partial P_1} + \lambda\left[\left(c_1 - P_1\right)\frac{\partial Q_1}{\partial P_1} - Q_1\right] = 0 \\
&\left(1-\alpha\right)\underline{h}_2 \frac{\partial Q_2}{\partial P_2} + \lambda\left[\left(c_2 - P_2\right)\frac{\partial Q_2}{\partial P_2} - Q_2\right] = 0 \\
&\left(c_1 - P_1\right)Q_1 + \left(c_2 - P_2\right)Q_2 = F
\end{aligned}
\tag{A2}
$$

Using Cramer's rule, we obtain[16]

$$
\frac{\partial P_1^*}{\partial F} = -\frac{1}{\left|H\right|}\frac{\partial^2 L_{NS}}{\partial {P_2}^2}\frac{\partial R}{\partial P_1}
\tag{A3}
$$

where $\left|H\right|$ is the Hessian of the system, which is constrained to be positive by the second-order condition for a maximum. Because the university sets net tuition below the profit-maximizing level, $\frac{\partial R}{\partial P_1} < 0$ over the relevant range, which means $\frac{\partial P_1^*}{\partial F}$ takes the sign of $\frac{\partial^2 L_{NS}}{\partial {P_2}^2}$.

Using (A1), $\frac{\partial^2 L_{NS}}{\partial {P_2}^2} = \left(1-\alpha\right)\underline{h}_2 \frac{\partial^2 Q_2}{\partial {P_2}^2} + \lambda\left[\left(c_2 - P_2\right)\frac{\partial^2 Q_2}{\partial {P_2}^2} - 2\frac{\partial Q_2}{\partial P_2}\right]$. Using substitutions from (1) and (A2), this reduces to $\lambda b_2\left(\gamma_2 + 1\right)\left(a_2 - P_2\right)^{\gamma_2 - 1} < 0$. ■

[16] Given symmetry, it is sufficient to derive comparative static results for group 1; corresponding conclusions may be drawn for both groups.

Proposition 2: With the university choosing both P_i and ϕ_i (i=1,2) to maximize (2) subject to (4), the Lagrangian function generalizes to

$$\begin{aligned} L_S &= \alpha h_1(\phi_1)\phi_1 Q_1(P_1) + (1-\alpha)h_2(\phi_2)\phi_2 Q_2(P_2) \\ &+\lambda\left[(c_1-P_1)\phi_1 Q_1(P_1) + (c_2-P_2)\phi_2 Q_2(P_2) - F\right] \end{aligned} \tag{A4}$$

The FOCs for an interior solution grow more complex, to wit,

$$\begin{aligned} &\alpha h_1\phi_1 \frac{\partial Q_1}{\partial P_1} + \lambda\left[(c_1-P_1)\phi_1\frac{\partial Q_1}{\partial P_1} - \phi_1 Q_1\right] = 0 \\ &(1-\alpha)h_2\phi_2 \frac{\partial Q_2}{\partial P_2} + \lambda\left[(c_2-P_2)\phi_2\frac{\partial Q_2}{\partial P_2} - \phi_2 Q_2\right] = 0 \\ &\alpha h_1 Q_1 + \alpha\sigma_1\phi_1 Q_1 + \lambda(c_1-P_1)Q_1 = 0 \\ &(1-\alpha)h_2 Q_2 + (1-\alpha)\sigma_2\phi_2 Q_2 + \lambda(c_2-P_2)Q_2 = 0 \\ &(c_1-P_1)\phi_1 Q_1 + (c_2-P_2)\phi_2 Q_2 = F \end{aligned} \tag{A5}$$

Using Cramer's rule and simplifying,

$$\begin{aligned} \frac{\partial P_1^*}{\partial F} &= \frac{1}{|H|}\left(\frac{\partial^2 L_S}{\partial P_1\partial\phi_1}\frac{\partial R}{\partial\phi_1} - \frac{\partial^2 L_S}{\partial\phi_1^2}\frac{\partial R}{\partial P_1}\right)\left(\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial\phi_2^2} - \left[\frac{\partial^2 L_S}{\partial P_2\partial\phi_2}\right]^2\right) \\ \frac{\partial\phi_1^*}{\partial F} &= -\frac{1}{|H|}\left(\frac{\partial^2 L_S}{\partial P_1^2}\frac{\partial R}{\partial\phi_1} - \frac{\partial^2 L_S}{\partial P_1\partial\phi_1}\frac{\partial R}{\partial P_1}\right)\left(\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial\phi_2^2} - \left[\frac{\partial^2 L_S}{\partial P_2\partial\phi_2}\right]^2\right) \end{aligned} \tag{A6}$$

where, again, $|H| > 0$ is the Hessian of the system. The second parenthetical expression is the same in each of the two equations, while the first parenthetical expression differs between the two.

All three expressions may be signed unambiguously. Using (A4) and substitutions from (A5), $\frac{\partial^2 L_S}{\partial\phi_2^2} = 2(1-\alpha)Q_2\sigma_2$ and $\frac{\partial^2 L_S}{\partial P_2\partial\phi_2} = -\lambda Q_2$. Comparing (A1) and (A4) reveals

$\frac{\partial^2 L_S}{\partial P_2^2} = \phi_1\frac{\partial^2 L_{NS}}{\partial P_2^2}$, therefore, using the proof of Proposition 1,

$$\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial \phi_2^2}-\left[\frac{\partial^2 L_S}{\partial P_2 \partial \phi_2}\right]^2=\left[\lambda b_2\phi_2\left(\gamma_2+1\right)\left(a_2-P_2\right)^{\gamma_2-1}\right]\left[2\left(1-\alpha\right)Q_2\sigma_2\right]-\lambda^2 Q_2^2$$
$$=\left[\frac{\lambda\phi_2\left(\gamma_2+1\right)}{a_2-P_2}\right]\left[2\left(1-\alpha\right)Q_2^2\sigma_2\right]-\lambda^2 Q_2^2$$

The FOCs (A5) give $\lambda=\frac{-\left(1-\alpha\right)\gamma_2 h_2}{\left(c_2-P_2\right)\gamma_2+\left(a_2-P_2\right)}$ and $\sigma_2\phi_2=\frac{-h_2\left(a_2-P_2\right)}{\left[\left(c_2-P_2\right)\gamma_2+\left(a_2-P_2\right)\right]}$,

therefore

$$\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial \phi_2^2}-\left[\frac{\partial^2 L_S}{\partial P_2 \partial \phi_2}\right]^2=\lambda\left(1-\alpha\right)Q_2^2\left\{\left[\frac{\gamma_2+1}{a_2-P_2}\right]\left[2\sigma_2\phi_2\right]+\frac{\gamma_2 h_2}{\left(c_2-P_2\right)\gamma_2+\left(a_2-P_2\right)}\right\}$$
$$=\lambda\left(1-\alpha\right)Q_2^2\left\{\left[\frac{\gamma_2+1}{a_2-P_2}\right]\left[\frac{-2h_1\left(a_1-P_1\right)}{\left[\left(c_1-P_1\right)\gamma_1+\left(a_1-P_1\right)\right]}\right]+\frac{\gamma_2 h_2}{\left(c_2-P_2\right)\gamma_2+\left(a_2-P_2\right)}\right\}$$
$$=-\lambda\left(1-\alpha\right)Q_2^2 h_1\left\{\frac{\left(\gamma_2+2\right)}{\left[\left(c_1-P_1\right)\gamma_1+\left(a_1-P_1\right)\right]}\right\}>0$$

This uses the fact that $\left(c_1-P_1\right)\gamma_1+\left(a_1-P_1\right)>0$, by the first FOC equation in (A5). Now, using symmetry, $\frac{\partial^2 L_S}{\partial \phi_1^2}=2\alpha\sigma_1 Q_1$ and $\frac{\partial^2 L_S}{\partial P_1 \partial \phi_1}=-\lambda Q_1$. Therefore, using (4) and (A4), and substitutions from (1) and (A5),

$$\frac{\partial^2 L_S}{\partial P_1\partial \phi_1}\frac{\partial R}{\partial \phi_1}-\frac{\partial^2 L_S}{\partial \phi_1^2}\frac{\partial R}{\partial P_1}=-\lambda Q_1^2\left(c_1-P_1\right)+\left[2\alpha\sigma_1 Q_1\right]\left\{\phi_1 Q_1\left[\frac{\gamma_1\left(c_1-P_1\right)+\left(a_1-P_1\right)}{a_1-P_1}\right]\right\}$$
$$=-\lambda Q_1^2\left(c_1-P_1\right)+-2\alpha\sigma_1\phi_1 Q_1^2\left[\frac{h_1}{\sigma_1\phi_1}\right]$$
$$=-\lambda Q_1^2\left(c_1-P_1\right)+-2\alpha h_1 Q_1^2$$
$$=-Q_1^2\left[\lambda\left(c_1-P_1\right)+2\alpha h_1\right]<-Q_1^2\left[\lambda\left(c_1-P_1\right)+\alpha h_1+\alpha\sigma_1\phi_1\right]=0$$

Thus, $\frac{\partial P_1^*}{\partial F}<0$.

It remains to show that $\frac{\partial^2 L_S}{\partial P_1^2}\frac{\partial R}{\partial \phi_1}-\frac{\partial^2 L_S}{\partial P_1\partial \phi_1}\frac{\partial R}{\partial P_1}>0$. Using $\frac{\partial^2 L_S}{\partial P_1^2}=\phi_1\frac{\partial^2 L_{NS}}{\partial P_1^2}$,

$$\frac{\partial^2 L_S}{\partial P_1^2}\frac{\partial R}{\partial \phi_1}-\frac{\partial^2 L_S}{\partial P_1 \partial \phi_1}\frac{\partial R}{\partial P_1}=\left[\lambda b_1\phi_1\left(\gamma_1+1\right)\left(a_1-P_1\right)^{\gamma_1-1}\right]\left(c_1-P_1\right)Q_1+\left(-\lambda Q_1\right)\phi_1 Q_1\left[\frac{\gamma_1\left(c_1-P_1\right)+\left(a_1-P_1\right)}{a_1-P_1}\right]$$

$$=\left[\frac{\lambda\left(\gamma_1+1\right)\phi_1 Q_1}{a_1-P_1}\right]\left(c_1-P_1\right)Q_1-\lambda Q_1^{\,2}\phi_1\left[\frac{\gamma_1\left(c_1-P_1\right)+\left(a_1-P_1\right)}{a_1-P_1}\right]$$

$$=\frac{\lambda Q_1^{\,2}\phi_1}{a_1-P_1}\left\{\left[\gamma_1+1\right]\left(c_1-P_1\right)-\left[\gamma_1\left(c_1-P_1\right)+\left(a_1-P_1\right)\right]\right\}$$

$$=\frac{\lambda Q_1^{\,2}\phi_1}{a_1-P_1}\left\{\left(c_1-P_1\right)-\left(a_1-P_1\right)\right\}=\frac{\lambda Q_1^{\,2}\phi_1}{a_1-P_1}\left(c_1-a_1\right)>0$$

Thus, $\frac{\partial \phi_1^*}{\partial F}<0$. ■

Proposition 3: Manipulating the FOCs (A5) using (1), one obtains $\alpha=\frac{-\lambda\left(c_1-P_1\right)}{h_1}-\frac{\lambda\left(a_1-P_1\right)}{h_1\gamma_1}$. $P_1=c_1$ implies $\alpha=-\frac{\lambda\left(a_1-c_1\right)}{h_1\gamma_1}$. If $\frac{\partial P_1^*}{\partial \alpha}<0$ can be demonstrated, then $\alpha<-\frac{\lambda\left(a_1-c_1\right)}{h_1\gamma_1}$ implies $P_1>c_1$, proving the claim about relative attractiveness. Using Cramer's rule,

$$\frac{\partial P_1^*}{\partial \alpha}=\frac{1}{|H|}\left\{\begin{array}{l}\left(\frac{\partial^2 L_s}{\partial \phi_1^{\,2}}\frac{\partial R}{\partial P_1}-\frac{\partial^2 L_s}{\partial P_1\partial \phi_1}\frac{\partial R}{\partial \phi_1}\right)\left\{\left[h_2+\sigma_2\phi_2\right]Q_2\left(\frac{\partial^2 L_s}{\partial P_2^{\,2}}\frac{\partial R}{\partial \phi_2}-\frac{\partial^2 L_s}{\partial P_2\partial \phi_2}\frac{\partial R}{\partial P_2}\right)-h_2\phi_2\frac{\partial Q_2}{\partial P_2}\left(\frac{\partial^2 L_s}{\partial P_2\partial \phi_2}\frac{\partial R}{\partial \phi_2}-\frac{\partial^2 L_s}{\partial \phi_2^{\,2}}\frac{\partial R}{\partial P_2}\right)\right\}\\ +\left[\left(h_1+\sigma_1\phi_1\right)Q_1\frac{\partial^2 L_s}{\partial P_1\partial \phi_1}-h_1\phi_1\frac{\partial Q_1}{\partial P_1}\frac{\partial^2 L_s}{\partial \phi_1^{\,2}}\right]\left[\frac{\partial R}{\partial P_2}\left(\frac{\partial^2 L_s}{\partial P_2\partial \phi_2}\frac{\partial R}{\partial \phi_2}-\frac{\partial^2 L_s}{\partial \phi_2^{\,2}}\frac{\partial R}{\partial P_2}\right)-\frac{\partial R}{\partial \phi_2}\left(\frac{\partial^2 L_s}{\partial P_2^{\,2}}\frac{\partial R}{\partial \phi_2}-\frac{\partial^2 L_s}{\partial P_2\partial \phi_2}\frac{\partial R}{\partial P_2}\right)\right]\\ +\left[\left(h_1+\sigma_1\phi_1\right)Q_1\frac{\partial R}{\partial P_1}\frac{\partial R}{\partial \phi_1}-h_1\phi_1\frac{\partial Q_1}{\partial P_1}\frac{\partial R}{\partial \phi_1}^{2}\right]\left(\frac{\partial^2 L_s}{\partial P_2\partial \phi_2}^{2}-\frac{\partial^2 L_s}{\partial P_2^{\,2}}\frac{\partial^2 L_s}{\partial \phi_2^{\,2}}\right)\end{array}\right\}$$

Manipulating the FOCs (A5) yields $\left(h_1+\sigma_1\phi_1\right)Q_1=-\frac{\lambda}{\alpha}\frac{\partial R_1}{\partial \phi_1}$ and $h_1\phi_1\frac{\partial Q_1}{\partial P_1}=-\frac{\lambda}{\alpha}\frac{\partial R_1}{\partial P_1}$. Using this,

$$\frac{\partial P_1^*}{\partial \alpha}=\frac{1}{|H|}\frac{\lambda}{(1-\alpha)\alpha}\left(\frac{\partial^2 L_s}{\partial \phi_1^2}\frac{\partial R}{\partial P_1}-\frac{\partial^2 L_s}{\partial P_1 \partial \phi_1}\frac{\partial R}{\partial \phi_1}\right)\left[\frac{\partial R}{\partial P_2}\left(\frac{\partial^2 L_s}{\partial P_2 \partial \phi_2}\frac{\partial R}{\partial \phi_2}-\frac{\partial^2 L_s}{\partial \phi_2^2}\frac{\partial R}{\partial P_2}\right)-\frac{\partial R}{\partial \phi_2}\left(\frac{\partial^2 L_s}{\partial P_2^2}\frac{\partial R}{\partial \phi_2}-\frac{\partial^2 L_s}{\partial P_2 \partial \phi_2}\frac{\partial R}{\partial P_2}\right)\right]$$

The first expression in parentheses is positive, from proof of Proposition 2. It remains to show that the expression in brackets is positive. Using (4) and substitutions from the proof of Proposition 2, the bracketed expression becomes

$$\begin{aligned}&-2\lambda Q_2\left[(c_2-P_2)Q_2\right]\left[(c_2-P_2)\phi_2\frac{\partial Q_2}{\partial P_2}-\phi_2 Q_2\right]\\&-\left[2(1-\alpha)Q_2\sigma_2\right]\left[(c_2-P_2)\phi_2\frac{\partial Q_2}{\partial P_2}-\phi_2 Q_2\right]^2+\left[\frac{\lambda\phi_2(\gamma_2+1)}{\gamma_2}\frac{\partial Q_2}{\partial P_2}\right]\left[(c_2-P_2)Q_2\right]^2\end{aligned}$$

The FOCs (A5) yield $\sigma_2\phi_2 Q_2=\frac{-\lambda}{1-\alpha}\left(Q_2^2 \Big/ \frac{\partial Q_2}{\partial P_2}\right)$; substituting this into the expression and rearranging obtains

$$\begin{aligned}&\frac{\lambda\phi_2 Q_2^2}{\frac{\partial Q_2}{\partial P_2}}\left\{\frac{\gamma_2+1}{\gamma_2}(c_2-P_2)^2\left(\frac{\partial Q_2}{\partial P_2}\right)^2-2(c_2-P_2)Q_2\frac{\partial Q_2}{\partial P_2}+2Q_2^2\right\}\\&>\frac{\lambda\phi_2 Q_2^2}{\frac{\partial Q_2}{\partial P_2}}\left\{(c_2-P_2)^2\left(\frac{\partial Q_2}{\partial P_2}\right)^2-2(c_2-P_2)Q_2\frac{\partial Q_2}{\partial P_2}+Q_2^2\right\}\\&=\frac{\lambda\phi_2 Q_2^2}{\frac{\partial Q_2}{\partial P_2}}\left[(c_2-P_2)\left(\frac{\partial Q_2}{\partial P_2}\right)-Q_2\right]^2>0\end{aligned}$$

Thus, $\frac{\partial P_1^*}{\partial \alpha}<0$.

Concerning the claim about limited funding, note that $|\lambda|$ sufficiently large and h_1 sufficiently small lead to $P_1>c_1$ even for large values of α. $\frac{\partial \phi_1^*}{\partial F}<0$ implies h_1 gets smaller as funding dwindles. One needs only to show $\frac{\partial \lambda^*}{\partial F} > 0$. By Cramer's rule, and using the proof of Proposition 2 and symmetry,

$$\frac{\partial \lambda^*}{\partial F} = \frac{1}{|H|}\left(\frac{\partial^2 L_S}{\partial P_1^2}\frac{\partial^2 L_S}{\partial \phi_1^2} - \left[\frac{\partial^2 L_S}{\partial P_1 \partial \phi_1}\right]^2\right)\left(\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial \phi_2^2} - \left[\frac{\partial^2 L_S}{\partial P_2 \partial \phi_2}\right]^2\right) > 0 . \ \blacksquare$$

Proposition 4: Define $T_i \equiv h_i \phi_i Q_i$ as the total quality units from enrolled students in group *i*. Then, (2) may be written $U = \alpha T_1 + (1-\alpha) T_2$. Examine the third and fourth equations in the FOCs (A5). If $P_i > c_i$, then the third term of the relevant equation is positive, implying $h_i Q_i + \sigma_i \phi_i Q_i = \frac{dT_i}{d\phi_i} < 0$. Thus, the marginal student from group *i* decreases the university's utility. ■

Proposition 5: Using (5) and (A6), and substitutions from the proof of Proposition 2,

$$\frac{\partial E_1}{\partial F} = \frac{1}{|H|}\left(\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial \phi_2^2} - \left[\frac{\partial^2 L_S}{\partial P_2 \partial \phi_2}\right]^2\right)\left[-\phi_1 \frac{\partial Q_1}{\partial P_1}\left(Q_1^2\left[\lambda(c_1 - P_1) + 2\alpha h_1\right]\right) - Q_1\left(\frac{\lambda Q_1^2 \phi_1}{a_1 - P_1}(c_1 - a_1)\right)\right]$$

Using $\alpha h_1 = \frac{-\lambda\left[(c_1 - P_1)\gamma_1 + (a_1 - P_1)\right]}{\gamma_1}$ from the FOCs (A5), this simplifies to

$$\frac{\partial E_1}{\partial F} = \frac{1}{|H|}\phi_1 \frac{\partial Q_1}{\partial P_1} Q_1^2 \lambda \left(\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial \phi_2^2} - \left[\frac{\partial^2 L_S}{\partial P_2 \partial \phi_2}\right]^2\right)\left(\frac{(\gamma_1 + 1)(c_1 - P_1) + (a_1 - P_1)}{\gamma_1}\right)$$

Given $\frac{\partial^2 L_S}{\partial P_2^2}\frac{\partial^2 L_S}{\partial \phi_2^2}-\left[\frac{\partial^2 L_S}{\partial P_2 \partial \phi_2}\right]^2 > 0$ from the Proof of Proposition 2, this expression takes the sign of $(\gamma_1+1)(c_1-P_1)+(a_1-P_1)$. Using (1), one can write $\varepsilon_1=\gamma_1 P_1/(a_1-P_1)$, therefore $\frac{\partial E_1}{\partial F}<0$ is equivalent to $\frac{P_1-c_1}{P_1}>\frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$. ■

Corollary 1: From the FOCs (A5), using $\varepsilon_1=\gamma_1 P_1/(a_1-P_1)$, one can write $\frac{P_1-c_1}{P_1}=\frac{1}{\lambda P_1}\alpha h_1+\frac{1}{\varepsilon_1}$. Solving together with $\frac{P_1-c_1}{P_1}=\frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$ yields $\alpha=-\frac{\lambda(a_1-c_1)}{h_1\gamma_1(\gamma_1+2)}$. Because $\frac{\partial P_1^*}{\partial \alpha}<0$, $\alpha<-\frac{\lambda(a_1-c_1)}{h_1\gamma_1(\gamma_1+2)}$ implies $\frac{P_1-c_1}{P_1}>\frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$. This proves the claim about relative attractiveness.

Note that $|\lambda|$ sufficiently large and h_1 sufficiently small lead to $\frac{P_1-c_1}{P_1}>\frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$ even for relatively large values of α. Therefore, $\frac{\partial \phi_1^*}{\partial F}<0$ and $\frac{\partial \lambda^*}{\partial F}>0$ prove the claim about limited funding. ■

Proposition 6: This follows from Proposition 2 by inspection of (5), given $Q_i>0$, $\frac{\partial Q_i}{\partial P_i}<0$, and $\phi_i>0$. ■

Appendix B. Effect on net tuition of an increase in demand

The effect that a change in an exogenous market variable, such as demand, has on net tuition and selectivity may be decomposed in a Slutsky-like manner.[17] For example, the effect on net tuition of a change in a variable x_1 may be written:

$$\frac{\partial P_i^*}{\partial x_1} = \frac{\partial P_i^D}{\partial x_1} - \frac{\partial R^*}{\partial x_1}\frac{\partial P_i^*}{\partial \bar{R}} \tag{A7}$$

The first term on the right-hand side of (A7) is the *substitution effect*, or direct effect of a change in x_1, holding utility constant. The second term is the *income effect*, or indirect effect of a change in x_1 via university resources. The first component in the income effect, $-\frac{\partial R^*}{\partial x_1}$, is the effect of a change in x_1 on the resource constraint, times -1. The second component, $\frac{\partial P_i^*}{\partial \bar{R}}$, is the effect on net tuition of loosening the resource constraint by \$1. Hence, $\frac{\partial P_i^*}{\partial \bar{R}} \equiv \frac{\partial P_i^*}{\partial F}$. It follows that the income effect of a change in exogenous condition x_1 is proportional to the effect of a positive funding shock, and the proportionality factor is $-\frac{\partial R^*}{\partial x_1}$.

Now let us use (A7) to analyze the effect of an isoelastic shift in demand, given by b_i in (1), on net tuition. Given $\partial \varepsilon_i / \partial b_i = 0$ and constant marginal cost, the substitution effect, $\frac{\partial P_i^D}{\partial b_1}$, is zero. Intuitively, if the demand shift does not cause demand elasticity or marginal cost to change, then no changes occur in the relative conditions between the two groups that would cause group *i*'s tuition to rise or fall. This means the effect of a demand shift is solely

[17] For further discussion on decomposing the effects of exogenous changes, see Varian (1984) and Nagler (2006).

the income effect, which is proportional to the effect of a positive funding shock.

Specifically,

$$\frac{\partial P_i^*}{\partial b_i} = -\frac{\partial P_i^*}{\partial \bar{R}} \frac{\partial R^*}{\partial b_i} = -\frac{\partial P_i^*}{\partial F} \frac{\partial R^*}{\partial b_i} = -\left(c_i - P_i\right) \phi_i \left(a_i - P_i\right)^{\gamma_i} \frac{\partial P_i^*}{\partial F} \qquad \text{(A8)}$$

This result may be verified directly using comparative static techniques.

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