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Presentation: Endogenous Mobility

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Comments
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Endogenous Mobility

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Acknowledgements and Disclaimer

$$\ln y_{it} = X_{it}\beta + \theta_i + \psi_{J(i,t)} + \epsilon_{it}$$

- **The Goal:** "Rehabilitate" the Abowd-Kramarz-Margolis decomposition
- **The Problem:** Structural interpretations rely on the assumption that job mobility is exogenous
- **The Approach:** Use the *realized mobility network* for structural estimation
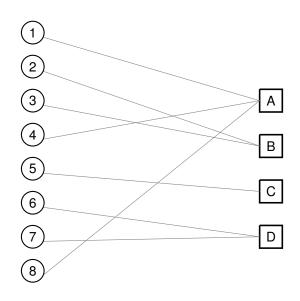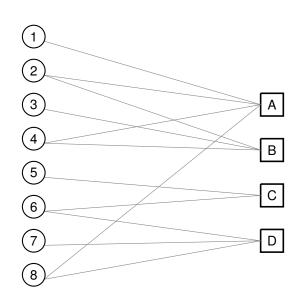
Estimating Individual and Employer Wage Effects

- The AKM (1999) specification for the wage determination equation with individual and employer heterogeneity

$$y = X\beta + D\theta + F\psi + \varepsilon$$

- where y is the $[N \times 1]$ stacked vector of log wage outcomes y_{it}, now sorted by t, then i.

- X is the $[N \times k]$ design matrix of observable individual and employer time-varying characteristics (the intercept is normally suppressed, with y and X measured as deviations from overall means).

- D is the $[N \times I]$ design matrix for the individual effects.

- F is the $[N \times J - 1]$ design matrix for the employer effects (non-employment is suppressed).

- ϵ is the $[N \times 1]$ vector of statistical errors, whose properties will be elaborated below.

- $\begin{bmatrix} \beta' & \theta' & \psi' \end{bmatrix}'$ are the unknown effects $[k \times 1]$, $[I \times 1]$, and $[J - 1 \times 1]$, resp., associated with each of the design matrices.

- Solving the fixed-effects moment equations

$$
\begin{bmatrix} X'X & X'D & X'F \\ D'X & D'D & D'F \\ F'X & F'D & F'F \end{bmatrix} \begin{bmatrix} \beta \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} X'y \\ D'y \\ F'y \end{bmatrix}
$$

  (then, imposing identification) yields estimates of the components of heterogeneity that can be used as the basis for consistent estimation of functions of the individual and employer effects.

- Next, we show how this specification relates to network models.

Labor Markets as Bipartite Graphs

- Imagine a set of I individuals, $A(t)$, and a set of J employers, $E(t)$ arranged in a bipartite graph where $A(t)$ and $E(t)$ are the two (disjoint) vertex (or node) sets

- There is a link between $i \in A(t)$ and $j \in E(t)$ if and only if i is employed by j at date t

- The totality of these links active at date t can be represented by the $I \times J$ adjacency matrix $B(t)$

- Assuming that we are modeling primary job holders, this adjacency matrix has a special form that will be critical in the modeling

- The labor market bipartite graph summarized by $B(t)$ evolves over time
- Since the employment relations between firms and workers can change at any time, it is reasonable to think of t as a continuous variable, sampled at intervals reflected in the data
- These considerations motivate adopting the dynamic network modeling tools to try to address the endogenous mobility issues

- We distinguish primary employment from other forms of employment
- The primary employer at time t is the current employer if there is only one
- Otherwise, the primary employer is the one to whom the individual supplies the most labor market time
- This assumption puts constraints on the row degree distribution of $B(t)$

- Specifically, assume that $j = 0$ refers to the non-employment state

- Including the column $j = 0$ ensures that every individual in the population at date t has exactly one "employer" although the (shadow) log wage outcome will be unobserved for individuals who are not employed at t

- Hence, $B(t) e_{J+1} = e_{J+1}$, where e_{J+1} is the $(J + 1) \times 1$ column vector of 1s

- Given this setup the column degree distribution, $e'_I B(t)$, is the size distribution of employers (technically only the columns 1 to J are included in this distribution)

- The employer size distribution (including not-employed) is also the column degree distribution of this bipartite graph

- We note that the (very hard) problem of entry and exit of individuals and employers can be included in this formalism by including columns in B for potential and defunct employers and allowing for birth and death of individuals. For the moment, we are not going to worry about this complication

- The existing data are snapshots of the labor market at points in time, $B(t_1), ..., B(t_T)$, where T is the total number of available time periods
- These adjacency matrices describe outcomes sampled at discrete points in time from the $I \times (J + 1)$ potential outcomes at each moment of time
- The objective is to use these snapshots of the labor market to test various assumptions about how the labor market evolves over time

Restating in Terms of the Adjacency Matrix Sequence

- Note that when the sort order is *t* then *i* we have:

$$F = \begin{bmatrix} B(1) \\ B(2) \\ \vdots \\ B(T) \end{bmatrix}$$

where $B(t)$ is the adjacency matrix from the bipartite labor market graph

- A direct strategy modeling endogenous mobility is to model the evolution of $B(t)$

- Workers, firms, and matches belong to latent heterogeneity classes
- a_i is the ability class of worker $i \in \{1, \ldots, I\}$.
- b_j is the productivity class of employer $j \in \{1, \ldots, J\}$.
- k_{ij} is the quality of the match between i and j
- Match quality depends on ability and productivity
- Earnings and mobility both depend on all three components
- **Wages:**

$$w_{ijt} = \alpha + a_i\theta + b_j\psi + k_{ij}\mu + \varepsilon_{ijt}$$

- **Mobility:** Probability of separation and transition depends on a, b and k.

- Observed Data

$$y_{it} = \left[w_{i\,\mathrm{J}(i,t)t}, s_{it}, i, \mathrm{J}(i,t) \right] \text{ for } i = 1, ..., I \text{ and } t = 1, ...T.$$

- Latent Data Vector:

$$Z = [a_1, \ldots, a_I, b_1, \ldots, b_J, k_{11}, k_{12}, \ldots, k_{1J}, k_{21}, \ldots, k_{IJ}]$$

- Parameter Vector:

$$\rho^T = \left[\alpha, \theta^T, \psi^T, \mu^T, \sigma, \gamma, \delta, \pi_a, \pi_b, \pi_{k|ab} \right], \rho \in \Theta$$

$$L\left(\rho \mid Y, Z\right) \quad \propto \quad \prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(w_{i\ J(i,t)t} - \alpha - a_i\theta - b_{J(i,t)}\psi - k_{i\ J(i,t)}\mu\right)^2}{2\sigma^2} \right] \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{J(i,t)} \rangle \langle k_{i\ J(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{J(i,t)} \rangle \langle k_{i\ J(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{J(i,t+1)} \rangle \mid \langle a_i \rangle \langle b_{J(i,t)} \rangle \langle k_{i\ J(i,t)} \rangle} \right]^{s_{it}} \end{array} \right\}$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} (\pi_{a\ell})^{a_{i\ell}} (\pi_{bm})^{b_{jm}} \left(\pi_{q \mid \ell m} \right)^{k_{ijq}} \right]$$

Gibbs Sampler Estimation of Posterior Distributions
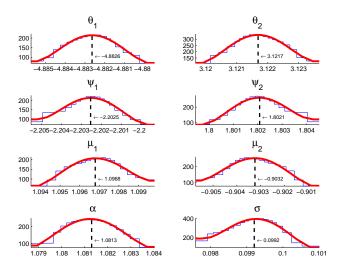
We use the Gibbs sampler to draw from $P(\rho, Z | Y)$

$$\sigma^{(1)} \sim p\left(\sigma | \alpha^{(0)}, \theta^{(0)T}, \psi^{(0)T}, \mu^{(0)T}, \gamma^{(0)}, \delta^{(0)}, \pi_a^{(0)}, \pi_b^{(0)}, \pi_{k|ab}^{(0)}, Z^{(0)}, Y\right)$$

$$\begin{bmatrix} \alpha \\ \theta \\ \psi \\ \mu \end{bmatrix}^{(1)} \sim p\left(\begin{bmatrix} \alpha \\ \theta \\ \psi \\ \mu \end{bmatrix} | \gamma^{(0)}, \delta^{(0)}, \pi_a^{(0)}, \pi_b^{(0)}, \pi_{k|ab}^{(0)}, Z^{(0)}, \sigma^{(1)}, Y\right)$$

$$\gamma^{(1)} \sim p\left(\gamma | \delta^{(0)}, \pi_a^{(0)}, \pi_b^{(0)}, \pi_{k|ab}^{(0)}, Z^{(0)}, \alpha^{(1)}, \theta^{(1)T}, \psi^{(1)T}, \mu^{(1)T}, Y\right)$$

$$\vdots$$

$$k_{IJ}^{(1)} \sim p\left(k_{IJ} | \rho^{(1)}, a_1^{(1)}, \ldots, a_I^{(1)}, b_1^{(1)}, \ldots, b_J^{(1)}, k_{11}^{(1)}, \ldots, k_{IJ-1}^{(1)}, Y\right)$$

	y	AKM θ	AKM ψ	AKM μ	AKM ε	Gibbs θ	Gibbs ψ	Gibbs μ	Gibbs ε	True θ	True ψ	True μ	True ε
AKM y	1												
θ	.879	1											
ψ	.393	-.069	1										
μ	.141	-.000	-.000	1									
ε	.024	0	0	0	1								
Gibbs θ	.867	.985	-.066	.000	0	1							
ψ	.395	-.062	.990	-.000	0	-.065	1						
μ	-.097	-.292	.183	.607	0	-.404	.166	1					
ε	.025	.001	-.002	.017	.960	-.000	-.000	.001	1				
True θ	.867	.985	-.066	.000	0	1	-.065	-.404	-.000	1			
ψ	.395	-.062	.990	-.000	0	-.065	1	.166	-.000	-.065	1		
μ	-.097	-.292	.183	.607	0	-.404	.166	1	.001	-.404	.166	1	
ε	.051	.020	.016	.017	.960	.020	.018	-.005	.999	.020	.018	-.005	1

Distribution of Wage Parameters: Simulated Data
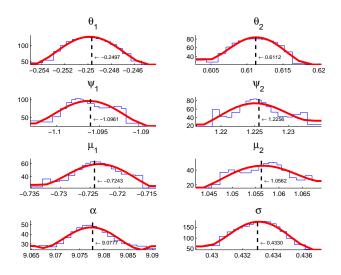
- Matched employer-employee data from LEHD program
- All individuals employed in IL, IN, WI between 1999-2003.
- 16.9 million persons
- 719 thousand unique employers
- 39 million unique person-employer matches
- Summaries of AKM decomposition (as described in Abowd, et al. [2003]) provide starting values and benchmarks.

- 0.25% simple random sample of individuals
- retain all matches and employers attached to those individuals
- 42,228 persons
- 39,458 employers
- 97,455 matches (including non-employment spells)
- 211,140 person-year observations

Posterior Distribution of Wage Equation Parameters: LEHD Data

Parameter	Mean	Std. Dev
θ_1	-0.2497	0.0032
θ_2	0.6112	0.0051
ψ_1	-1.0961	0.0044
ψ_2	1.2256	0.0055
μ_1	-0.7243	0.0066
μ_2	1.0562	0.0082
α	9.0777	0.0082
σ	0.4330	0.0024

Correlation Matrix of Wage Parameters: LEHD Data

		AKM				Gibbs			
		y	θ	ψ	μ	ε	θ	ψ	μ	ε
AKM	y	1								
	θ	0.5284	1							
	ψ	0.5683	0.0632	1						
	μ	0.4236	0.0335	-.0182	1					
	ε	0.2345	-.0000	-.0000	0.0000	1				
Gibbs	θ	0.3361	0.2401	0.1682	0.0816	-.0000	1			
	ψ	0.5486	0.2037	0.5599	0.1179	-.0000	0.0359	1		
	μ	-.02219	0.0951	-.2577	0.1396	0.0000	-.1202	-.7236	1	
	ε	0.4989	0.2288	0.1498	0.2677	0.4703	-.0000	.0002	-.0000	1

	θ_{Gibbs}	ψ_{Gibbs}	μ_{Gibbs}	ε_{Gibbs}
θ_{AKM}	0.151	0.317	0.154	0.175
ψ_{AKM}	0.1441	1.492	-.529	0.164
μ_{AKM}	0.072	0.332	0.266	0.307
ε_{AKM}	0.000	0.000	0.000	0.988
Constant	0.014	0.185	-.091	-.003

θ Type	ψ Type	μ Type	Separation Probability	
			AKM	Gibbs
1	1	1	0.4127	0.6603
1	1	2	0.0530	0.4418
1	2	1	0.0169	0.2796
1	2	2	0.0055	0.2506
1	3	-	0.0552	0.3128
2	1	1	0.7109	0.3814
2	1	2	0.0448	0.1799
2	2	1	0.0103	0.1923
2	2	2	0.0123	0.2373
2	3	-	0.1095	0.3179
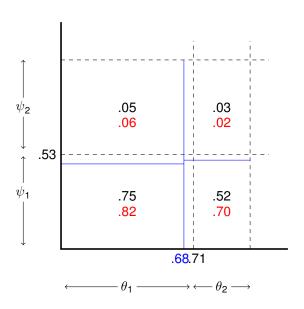
Destination Probabilities, δ: AKM and Structural Estimates

Origin			Destination Employer Type					
θ Type	ψ Type	μ Type	AKM			Gibbs		
			1	2	–	1	2	–
1	1	1	0.4103	0.1440	0.4457	0.4546	0.1129	0.4324
1	1	2	0.4758	0.2057	0.3185	0.4929	0.2160	0.2911
1	2	1	0.2315	0.3990	0.3695	0.2486	0.4681	0.2834
1	2	2	0.2205	0.4791	0.3004	0.0910	0.2815	0.6275
1	3	-	0.6912	0.3088	–	0.7247	0.2753	–
2	1	1	0.5000	0.1988	0.3012	0.5079	0.0319	0.4602
2	1	2	0.4929	0.2622	0.2449	0.5226	0.2324	0.2450
2	2	1	0.2178	0.5139	0.2683	0.1120	0.6474	0.2406
2	2	2	0.1660	0.5727	0.2613	0.1089	0.6242	0.2671
2	3	-	0.6189	0.3811	–	0.7229	0.2770	–

θ Type			1	1	2	2	3
	ψ Type		1	1	2	2	3
		μ Type	1	2	1	2	-
1	1	1	0.4127	0.2272	0.0708	0.0038	0.2855
1	1	2	0.0530	0.7230	0.0906	0.0048	0.1286
1	2	1	0.0169	0.0526	0.8447	0.0066	0.0792
1	2	2	0.0055	0.0173	0.0670	0.7530	0.1572
1	3	-	0.0552	0.1715	0.0818	0.0043	0.6872
2	1	1	0.7109	0.1014	0.0118	0.0004	0.1755
2	1	2	0.0448	0.8693	0.0405	0.0013	0.0441
2	2	1	0.0103	0.0113	0.9283	0.0038	0.0463
2	2	2	0.0123	0.0135	0.1436	0.7672	0.0634
2	3	-	0.1095	0.1202	0.0854	0.0027	0.6822

- Showed that endogenous mobility affects the AKM decomposition via the realized mobility network, which is the tool used for identification in that model

- Developed a complete posterior predictive distribution for incorporating endogenous mobility into the AKM wage decomposition

- The Markov transition matrix that describes the evolution of the network adjacency matrix reveals that the probability of transitions into better matches do depend on the worker type, firm type and match type in the current job

- Future work will refine the regression-based approach we used here for estimating the expected structural effect given the AKM wage components